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Symmetry groups of Moroccan geometric woodwork patterns

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Many works report the classification and analysis of geometric patterns, particularly those found in the Alhambra, Spain, but few authors have been interested in Moroccan motifs, especially those made on wood. Studies and analyses made on nearly a thousand Moroccan patterns constructed on wood and belonging to different periods between the 14th and 19th centuries show that, despite their great diversity, only five plane groups are present. Groups *p*4*mm* and *c*2*mm* are predominant, *p*6*mm* and *p*2*mm* are less frequent, while *p*4*gm* is rare. In this work, it is shown that it is possible to obtain the 17 plane symmetry groups by using a master craftsmen's method called *Hasba*. The set of patterns are generated from *n*-fold rosettes, considered as the basic motif, by the *Hasba* method. The combination and the overlap between these basic elements generate other basic elements. By repeating these basic elements, it is possible to construct patterns having various symmetry groups. In this article, only uncoloured patterns are considered and the interlace patterns are disregarded.

1. Introduction

Symmetry analysis of decorations and designs of various cultures was undertaken in the early part of the 20th century by several authors (Polya, 1924; Speiser, 1927). Some of them had a particular interest in the Arab–Islamic ornamental patterns. The symmetry groups of geometric patterns found in the Alhambra in Granada (Spain) have attracted the attention of mathematicians as well as crystallographers (Muller, 1944; Makovicky & Makovicky, 1977). Analysis of the Alhambra patterns raised controversy about the existence of 17 symmetry groups (Grünbaum & Grünbaum, 1986).

All these authors focused their work on classification and analysis of the patterns constructed in the Alhambra by using a set of characteristic shapes handcut from ceramic tiles called *zellijs* (Makovicky & Fenoll Hach-Ali, 1997; Fenoll Hach-Alí & López Galindo, 2003). Except for Castera (1999) and Makovicky & Makovicky (2011), very few authors have been interested in the symmetry of Moroccan patterns made on wood.

Moroccan geometric art (*Tasstir*) flourished in the 11th century and culminated under the Marinid dynasty (13th–14th century), as shown in masterpieces like the Madrasa Attarine (Fez). Engraved or painted wood used in traditional decoration is perfectly suited to the requirements of modern architecture. Sculpted or painted motifs exquisitely adorn the ceilings and the portals of mausoleums, historic monuments and luxurious private residences.

In a collection of patterns found mainly in Marrakech and Fez monuments, we identified nearly a thousand patterns belonging to different periods between the 14th and 19th centuries. These patterns were constructed by a method called *Hasba* (measure) described previously (Thalal *et al*, 2011). The *Hasba* method widely adopted by Moroccan craftsmen, especially those working in wood, leads to sophisticated panels constituted of motifs with central rosettes of symmetry 8×2^p and 12×2^p where $p = 0, 1, 2, 3$ and $5 \times 2^{p'}$ ($p' = 0, 1, 2$), surrounded by a region called the belt. The rosette, which is the most important element of the motif, predetermines the repeat pattern to be created.

Our investigation of Moroccan patterns worked in wood shows that, despite their great diversity, only five plane groups of symmetry are present. Groups *p*4*mm* and *c*2*mm* are predominant, *p*6*mm* and *p*2*mm* are less frequent, while *p*4*gm* is rare (Fig. 1). The absence of the other symmetry groups can be interpreted either by the fact that the craftsmen have reached the limit of their ability to perform other groups (they have no knowledge of symmetry groups or they did not have highly developed tools) or because they had a preference for certain symmetries and had omitted the others.

The present work shows that it is possible to obtain the 17 plane groups of symmetry generated from *n*-fold rosettes by the *Hasba* method. By repeating the rosettes, considered as the basic motif, we can make up patterns having various symmetry groups. The examples of the 17 symmetry groups presented in this article were obtained by tiling the plane successively by tenfold and 12-fold rosettes. Only uncoloured patterns are considered here and the interlace patterns were disregarded.

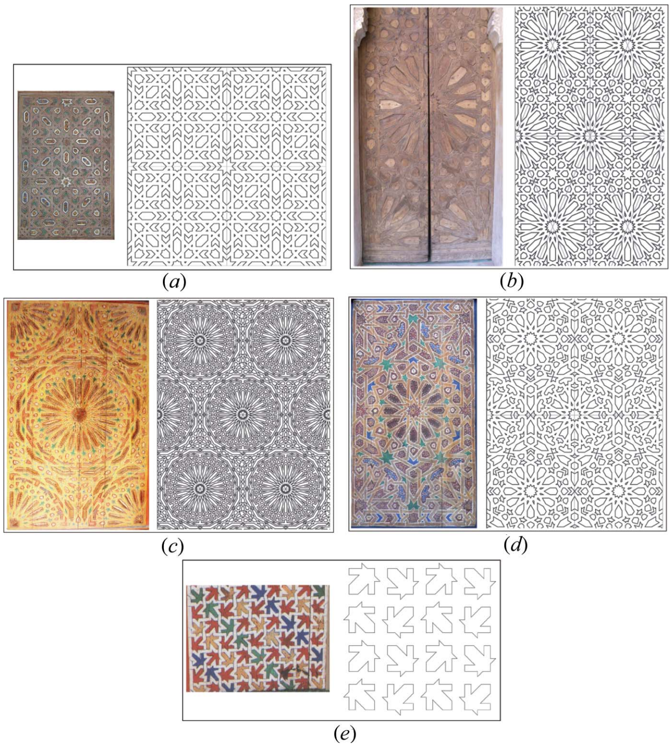


Figure 1
(*a*) Symmetry group *p4mm* from the Qaraouyine mosque in Fez (authors), (*b*) symmetry group *c2mm* from the Medersa Ben Youssef in Marrakech (authors), (*c*) symmetry group *p6mm* from the Bahia Palace in Marrakech (authors), (*d*) symmetry group *p2mm* from a private house in Marrakech (authors) and (*e*) symmetry group *p4mg* from the Dar Bieda in Marrakech (Paccard, 1980).

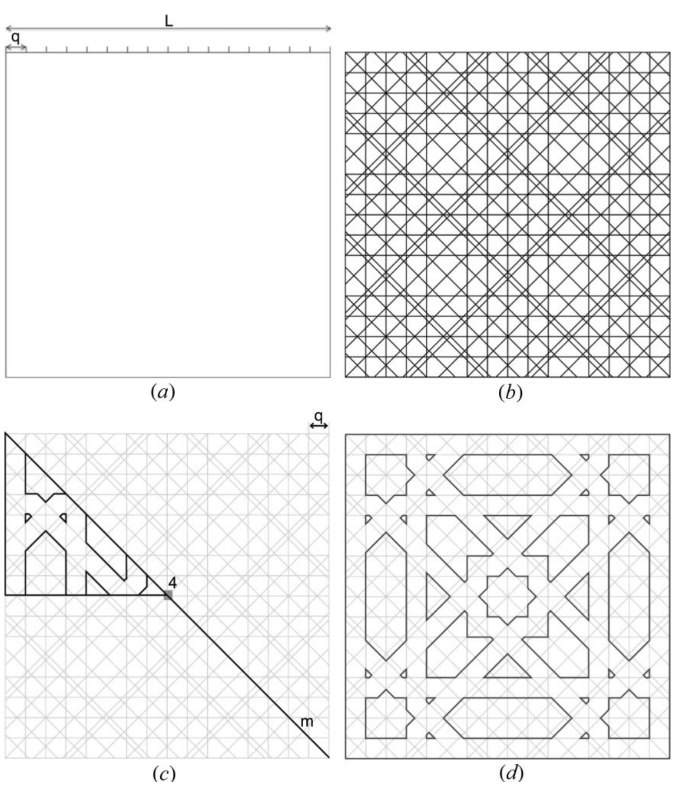


Figure 2
(*a*) Unit division on the square frame. (*b*) Grid with h = 16. (*c*) and (*d*) Construction of the pattern from the asymmetric unit.

2. Construction of tenfold and 12-fold rosettes by the *Hasba* method

The *Hasba* is the method of construction of geometric patterns that is the most used by craftsmen. It was described exhaustively by Thalal *et al.* (2011). Craftsmen start their work by drawing the general frame of the pattern, which is often square; rectangular, octagonal and others polygonal designs are not uncommon. On the sides of the frame they define an empirical unit division q. The sides of the square have thus a width L equal to a multiple of q: $L = hq$. The ratio h is the specific measure or '*Hasba*' of the pattern. The type of the pattern achieved depends strongly on h, which may be an integer or rational number greater than eight (Fig. 2*a*)

For a given value of h of the '*Hasba*', the pattern from a specific underlying grid constituted by four sets of crossed parallel lines (Figs. 2*b*, 2*c* and 2*d*) is drawn. The sets are related two-by-two by the fourfold axis rotations located at the centre of the square, mirror reflections in lines joining the midpoint of its sides and reflections in its diagonal.

For a high value of h, the '*Hasba*' leads to sophisticated patterns containing a central n-fold rosette. This is an n-star shape, where n is the number of rays.

The n-fold rosette, for instance $n = 12$, is the most important element of the pattern; it predetermines the design to be achieved. For this reason the pattern is named on the basis of its rosette symmetry.

We describe briefly here the process of constructing the pattern with a 12-fold rosette, which allows the eight symmetry groups generated below to be obtained.

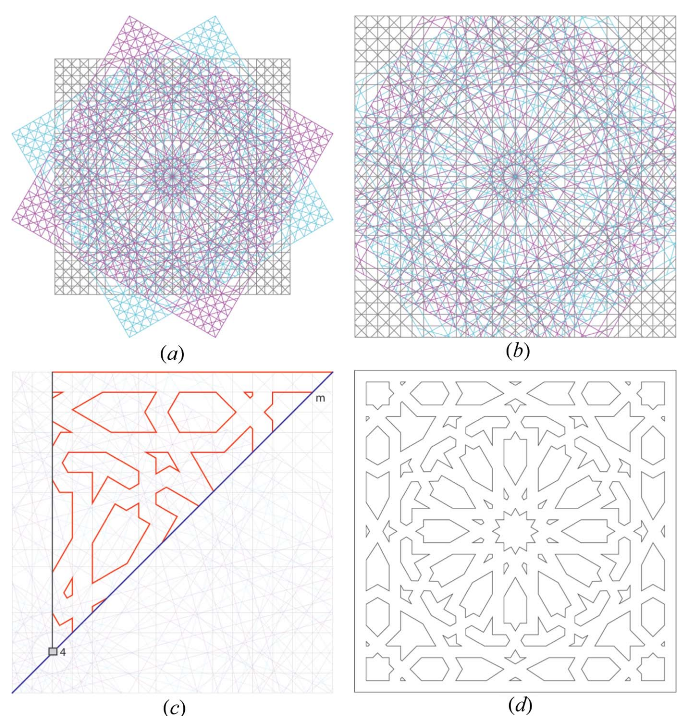


Figure 3
(*a*) Three superposed grids. (*b*) 12-grid. (*c*) Zoomed common area. (*d*) 12-fold pattern.

We start by tracing the first grid with $h = 28$; we superpose on it successively two identical grids at 30° to one another. The rotation axis is located at the centre of the grids (Fig. 3*a*). The result is a new grid, a so-called 12-grid (Fig. 3*b*).

We then trace on the lines in coincidence a part of the central rosette (Fig. 3*c*) and some elements of the pattern. Finally we construct the entire pattern by applying a mirror and a fourfold axis (Fig. 3*d*).

Patterns obtained by the *Hasba* method have interlaced ribbons whose width is equal to the unit measure q defined above. Even hidden in the constructed patterns, interlaced ribbons are naturally present in the design; extending the edges of the different shapes that constitute the patterns reveals them. Indeed, by marking out the hidden ribbon more artistic interlaced patterns emerge from the initial pattern. The method imposes that ribbons must be infinitely continuous and their width must be constant within the pattern as well as in every repeat pattern (Fig. 4). This is the main characteristic of the *Hasba*.

Other n-fold rosettes (where $n \geq 5$) can be obtained by the *Hasba* method. They constitute the fundamental elements used in the generation of the 17 symmetry groups. In this article, we shall use the tenfold (Fig. 5) and 12-fold rosettes to build the 17 groups.

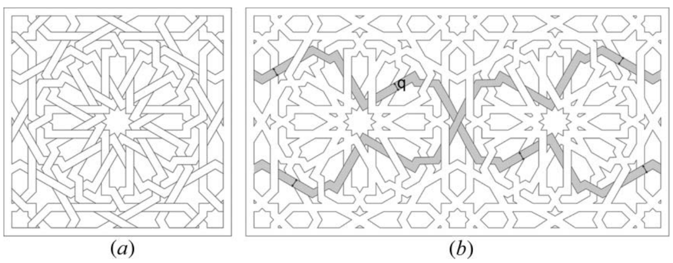


Figure 4
(*a*) Revealed ribbons: interlace pattern. (*b*) Infinite continuity of ribbons in a 12-fold pattern.

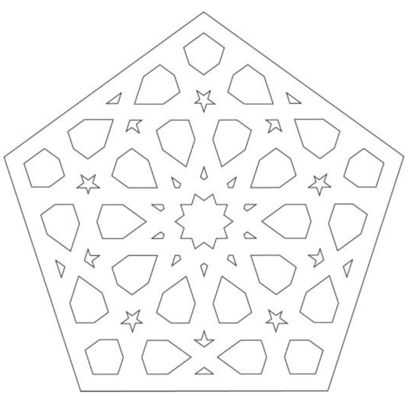


Figure 5
(*a*) Tenfold pattern.

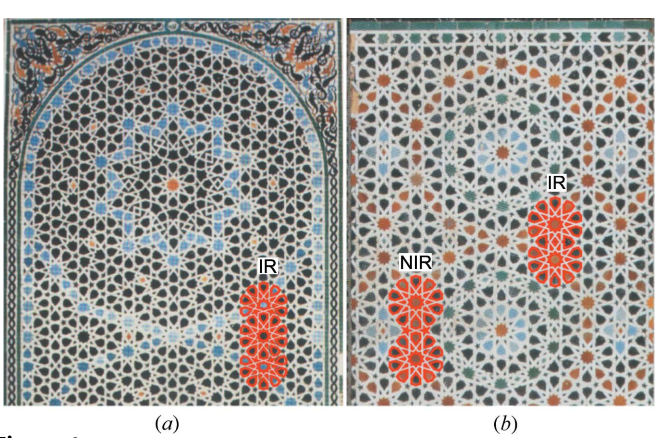


Figure 6
(*a*) Tenfold rosettes from the Medersa Attarine in Fez (Sijelmassi, 1991). (*b*) Tenfold rosettes from Medersa Bou Inania in Fez (Sijelmassi, 1991).

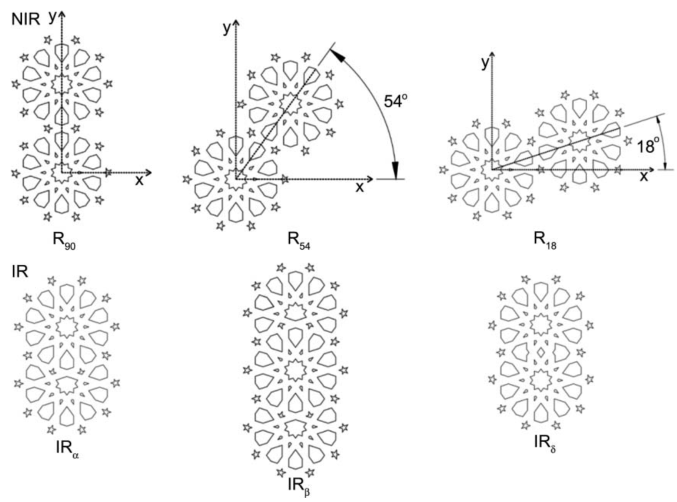


Figure 7
Not interpenetrated (NIR) and interpenetrated (IR) basic elements.

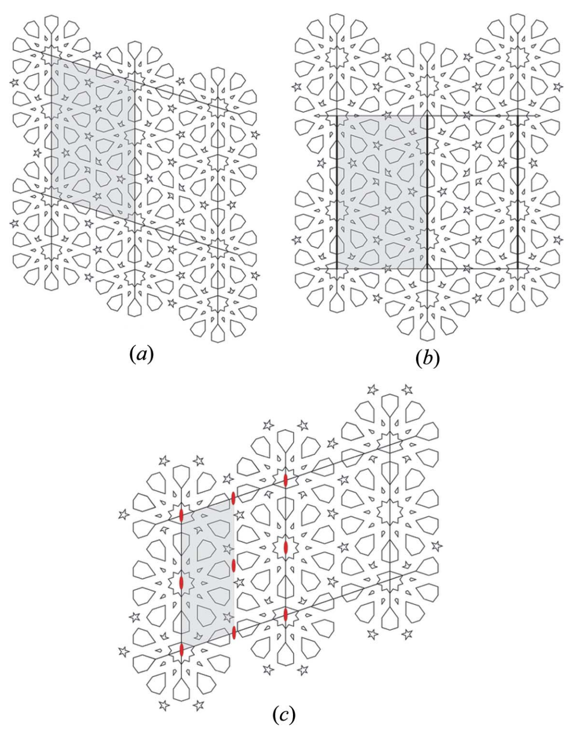


Figure 8
(*a*) *p*1, (*b*) *p*1*m* and (*c*) *p*2 symmetry groups.

3. Generation of the 17 plane groups of symmetry

The 17 symmetry groups are generated from the rosettes constructed by the *Hasba* method. To describe the generating of the groups, we chose two families of patterns. The first family is conceived from a tenfold rosette and the second from a 12-fold rosette.

3.1. Symmetry groups obtained from the tenfold rosette

A Moroccan pattern containing tenfold rosettes consists in repetition of interpenetrated (IR) and not interpenetrated (NIR) rosettes, as shown in Fig. 6.

The tenfold rosettes are always arranged according to six configurations or their combinations. In each configuration, the *Hasba* rule imposes that ribbons must be infinitely continuous and their width must be constant and equal to the unit division q.

Let x,y be the reference axes of the plane. The NIR oriented at 90, 54 and 18° relative to the x axis will be called R_{90}, R_{54} and R_{18}, respectively. The three IR oriented at 90° relative to the x axis will be called IR_{α}, IR_{β} and IR_{δ} (Fig. 7).

The initial tenfold pattern, from which we extracted the oblique decorations used to generate the symmetry groups, limits the values of the angles to 54 and 18°. These angles can not be chosen arbitrarily, otherwise the continuity condition imposed by the *Hasba* is no longer respected in the patterns to be constructed. For other symmetries of the rosettes, the angles will be obviously different.

These configurations are considered as the basic elements that contain the minimal geometrical information necessary to generate the symmetry group of the pattern.

The tiling of the plane with these basic elements or their combinations gives rise to patterns containing symmetry elements that determine the symmetry group of the pattern.

3.1.1. Groups *p*1, *p*1*m* and *p*2. The tiling of the plane with the basic element IR_{α} gives a pattern without any symmetry element except for translation and leads to symmetry group *p*1. By applying a mirror to the unit cell of the pattern *p*1, we obtain the group *pm*. In addition, the tiling with IR_{β} gives a pattern having a twofold axis and generates symmetry group *p*2 (Fig. 8).

3.1.2. Groups *c*2*mm*, *c*1*m*, *p*2*mg* and *p*2*gg*. The combination of the basic elements R_{90} and R_{54}, gives an aggregate of six rosettes having a gap at the centre as shown in Fig. 9(*a*). By linking the petals of the rosettes according to the rules of the *Hasba* method, we can fill this gap in several ways (Figs. 9*b*, 9*c* and 9*d*). By cutting out their cores and discarding the rest we

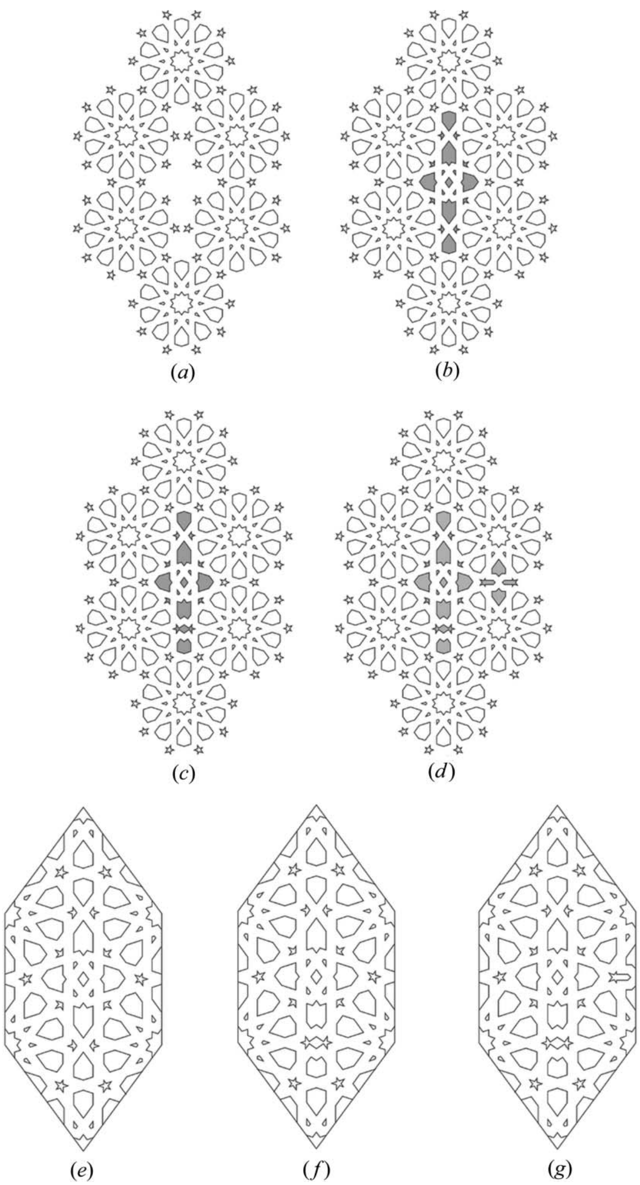


Figure 9
(*a*) Combination of the basic elements R_{90} and R_{54}. (*b*) Motif with the point group *mm*2. (*c*) Motif with the point group *m*. (*d*) Motif without any symmetry element. (*e*), (*f*) and (*g*) unit tiles (UT).

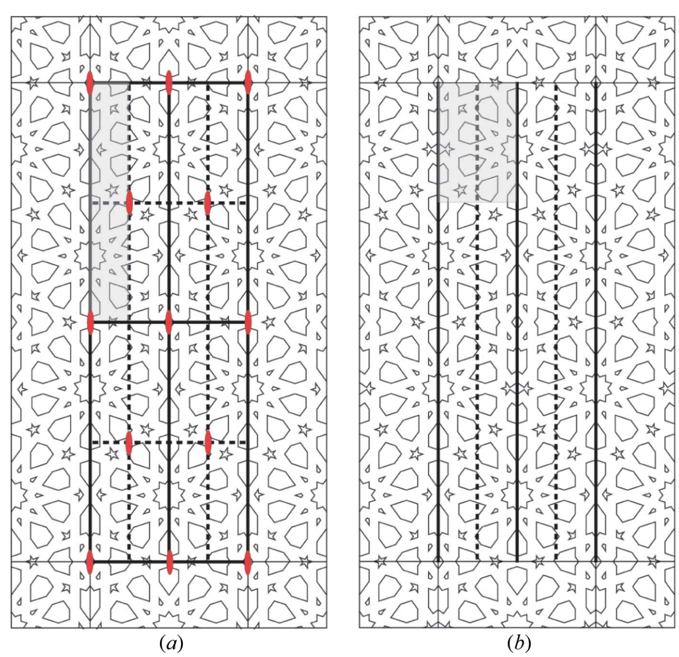
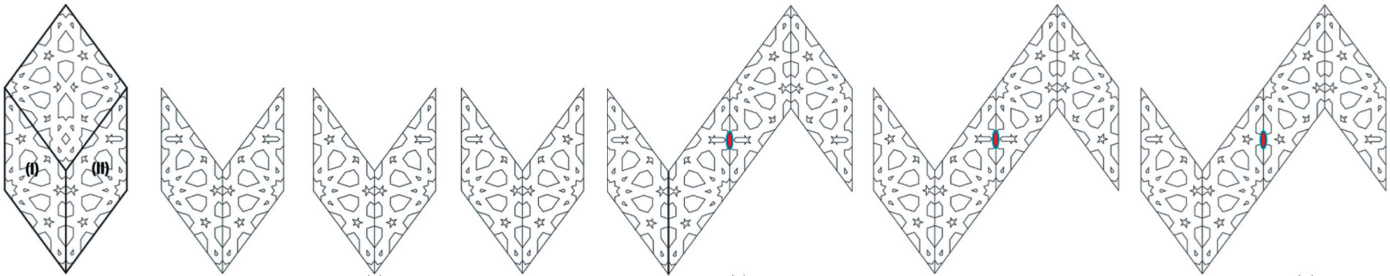
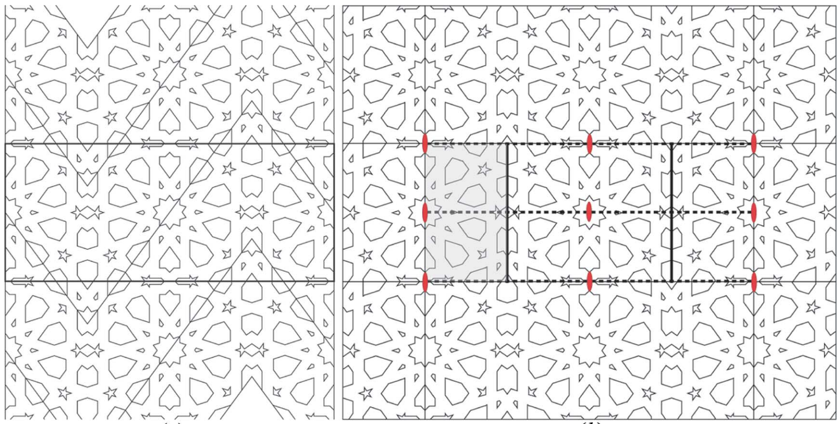


Figure 10
(*a*) The pattern of the group *c*2*mm*. (*b*) The pattern with the plane group *c*1*m*1.



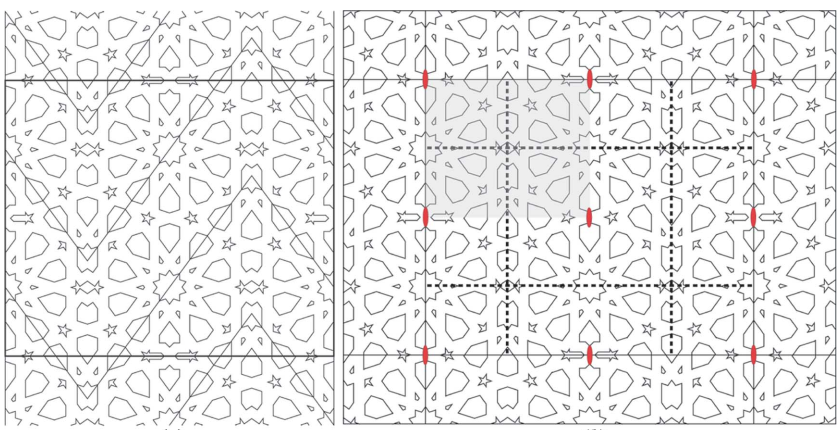
(*a*) (*b*) (*c*) (*d*) (*e*) (*f*) (*g*)

Figure 11
(*a*) Element UT_3 divided into three rhombs. (*b*), (*c*) and (*d*) three motifs obtained from UT_3. (*e*), (*f*) and (*g*) three friezes FR_1, FR_2 and FR_3.



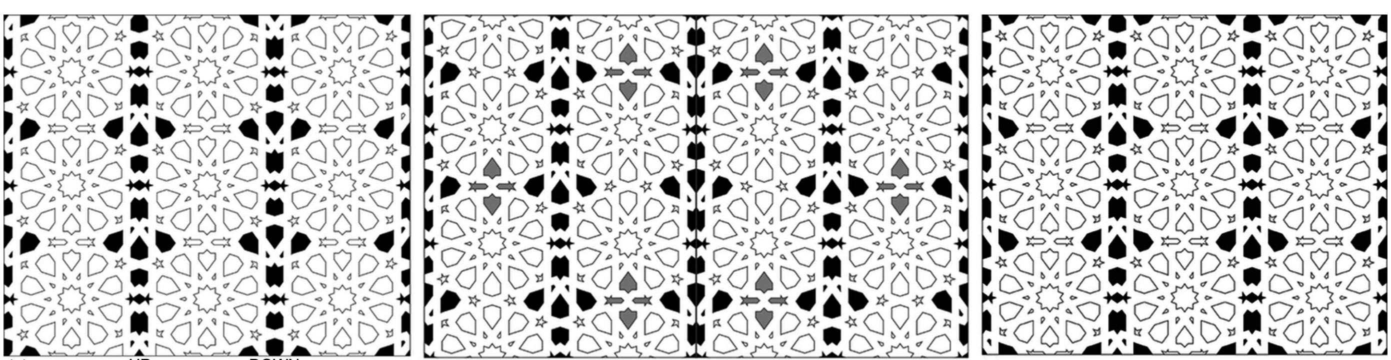
(*a*) (*b*)

Figure 12
(*a*) The tiling with FR_1. (*b*) The pattern with the plane group *p*2*mg*.



(*a*) (*b*)

Figure 13
(*a*) The zigzag aggregates RF_2–RF_3. (*b*) The pattern with the plane group *p*2*gg*.



(*a*) (*b*) (*c*)

Figure 14
(*a*) The thin slab up and down. (*b*) Twin of *pgg* structures. (*c*) The plane group *p*1*m*1.

obtain three unit tiles called UT_1, UT_2 and UT_3 as shown in Figs. 9(*e*), 9(*f*) and 9(*g*). The unit tiles UT_1 and UT_2 reveal two perpendicular mirrors (the point group *mm*2) and a single mirror plane (the point group *m*), respectively, while the unit tile UT_3 does not have any element of symmetry.

The tiling with UT_1 and UT_2 generates the *c*2*mm* and *c*1*m*1 patterns (Fig. 10), respectively.

The element UT_3 can be divided in three rhombs (Fig. 11*a*). From rhombs I and II alone, we can construct three motifs. The first one is obtained by applying a vertical mirror on rhomb II (Fig. 11*b*), the second one is formed by both rhombs I and II (Fig. 11*c*), and its image relative to a vertical mirror gives the last one (Fig. 11*d*). By applying a twofold axis, they give, respectively, three friezes FR_1 (Fig. 11*e*), FR_2 (Fig. 11*f*) and FR_3 (Fig. 11*g*) with frieze groups *pmg*.

Tilings with FR_1 generate the *p*2*mg* pattern (Fig. 12). The zigzag aggregates FR_2–FR_3 give the *p*2*gg* pattern (Fig. 13).

In Figs. 12 and 13, patterns are constructed by rosette-based slabs attached by thin slabs which have only symmetry *m*. The orientations of the thin slab (up or down) give different plane groups (Fig. 14*a*). This is the result of the fact that the rosette-based slab has higher frieze symmetry than required by the structure. So, in Fig. 13, the plane group *pgg* does not show the vertical *m* planes of the rosette-based slabs. The planes are not valid for the entire structure: they are only local. If the vertical *m* plane is used once, we get a twin of *pgg* structures (Fig. 14*b*). If it is used every time, starting in Fig. 13, and the thin slabs are still only *m*, we get the plane group *p*1*m*1 (Fig. 14*c*). So hypersymmetry of the rosettes leads to twinning or to other plane groups if occurring regularly.

3.1.3. Groups *p*2*mm* and *p*1*g*1. Following the same principle of construction described in the previous section, the combination of R_{54} and IR_δ gives two new motifs (Figs. 15*a* and 15*b*), from which we extract two unit tiles UT_4 (Fig. 15*b*) and UT_5 (Figs. 15*c* and 15*d*).

The tiling with UT_1 followed by a row of the element UT_4, and so on, leads to pattern *p*2*mm* (Fig. 16*a*). The tiling with UT_5 leads to pattern *p*1*g*1 (Fig. 16*b*).

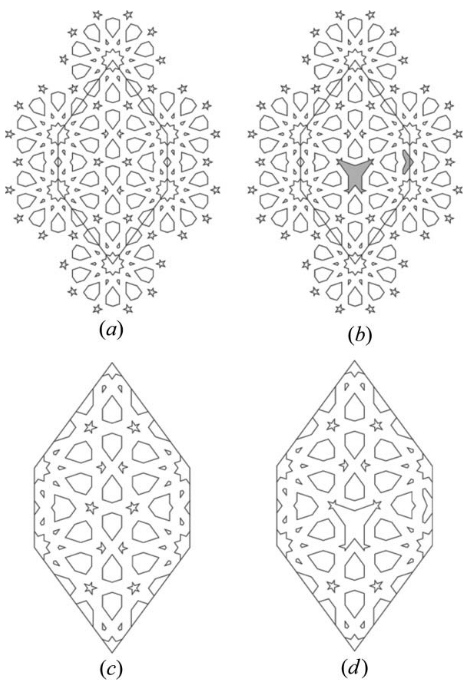


Figure 15
New unit tiles (*b*) and (*d*) extracted from motifs (*a*) and (*c*), respectively.

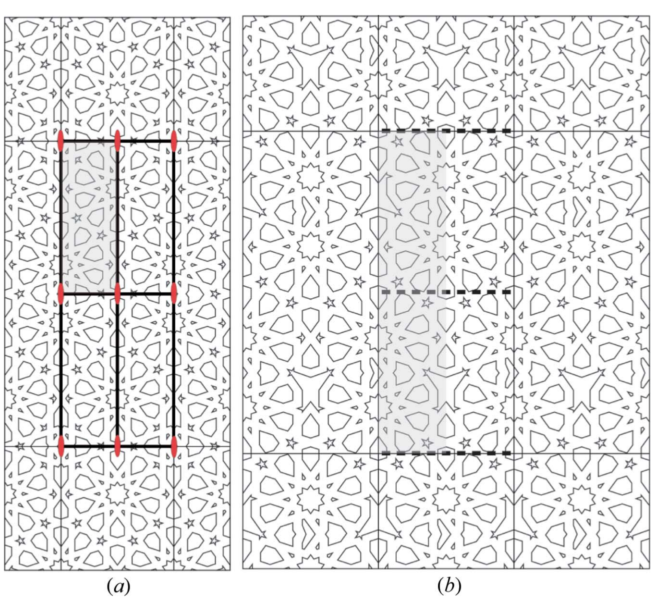


Figure 16
(*a*) The pattern of the group *p*2*mm*. (*b*) The pattern with the plane group *p*1*g*1.

3.2. Symmetry groups obtained from 12-fold rosettes

3.2.1. Group *p*4*mm* and *p*4*gm*. By using the *Hasba* method, we construct two square tiles with two types of 12-fold central rosettes called unit tiles UT_6 and UT_7 (Fig. 17*a*). The tiling with UT_6 or UT_7 generates the *p*4*mm* pattern (Fig. 17*b*).

A combination of $\frac{2}{3}UT_6$ and $\frac{1}{3}UT_7$ generates a hybrid tile HT_1 (point group 4*mm*) (Fig. 18*a*). The tiling based on HT_1 gives the *p*4*gm* pattern (Fig. 18*b*).

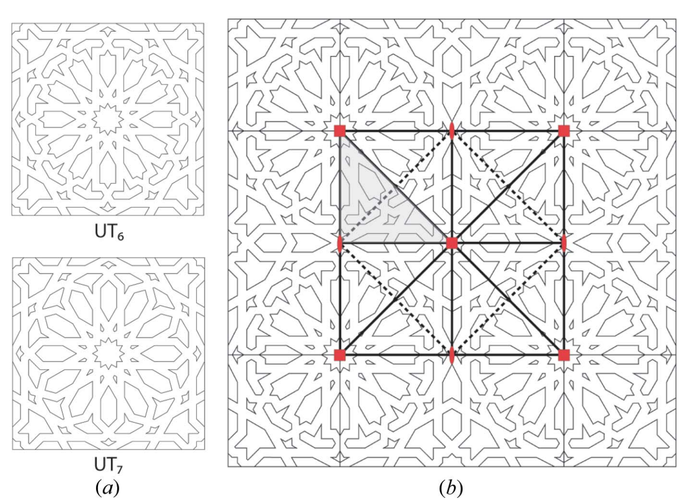


Figure 17
(*a*) Unit tiles UT_6 and UT_7. (*b*) The pattern with the plane group *p*4*mm*.

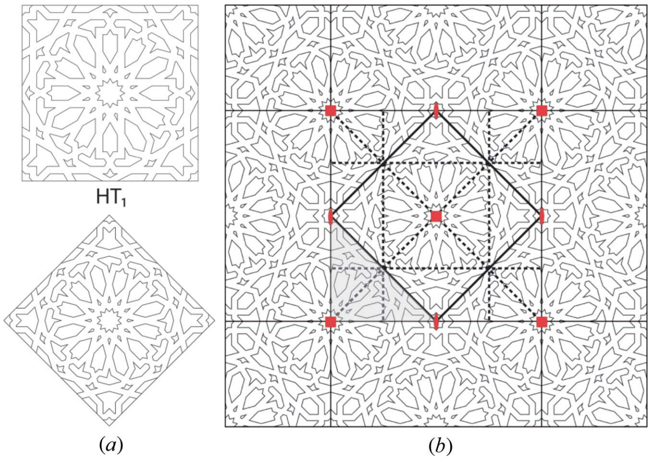


Figure 18
(*a*) The hybrid unit tile HT$_1$ rotated by an angle of 45°. (*b*) The pattern with the plane group *p*4*gm*.

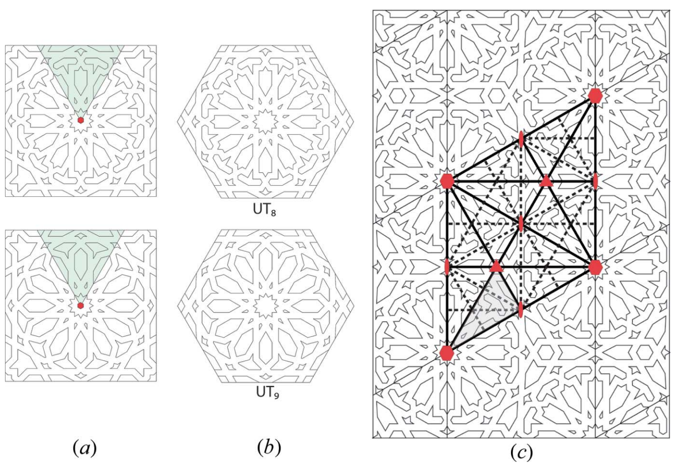


Figure 19
(*a*) The template with a shape of equilateral triangles. (*b*) The hexagonal tiles UT$_8$ and UT$_9$. (*c*) The pattern with the plane group *p*6*mm*.

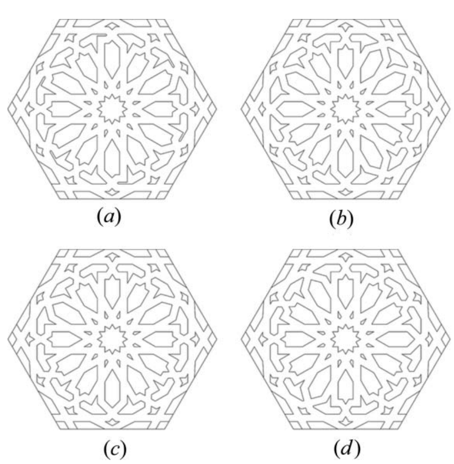


Figure 20
The hybrid tiles (*a*) HT$_2$, (*b*) HT$_3$, (*c*) HT$_4$ and (*d*) HT$_5$ with point groups 6, 3 and 3*m*.

3.2.2. Groups *p*6*mm*, *p*6, *p*3, *p*3*m*1 and *p*31*m*. From the unit tiles UT$_6$ and UT$_7$, we extract a template having the shape of equilateral triangles. By rotation about a sixfold axis passing through the apex of the triangles, we obtain two hexagonal tiles UT$_8$ and UT$_9$, as shown in Figs. 19(*a*) and 19(*b*). The tiling with one of them gives a *p*6*mm* pattern (Fig. 19*c*).

The combination of the hexagonal tiles UT$_8$ and UT$_9$ in different proportions generates a hybrid tile HT$_2$ (Fig. 20*a*), which has point group six, and three other hybrid tiles HT$_3$, HT$_4$ and HT$_5$ (Figs. 20*b*, 20*c* and 20*d*), with a threefold axis at the centre, which differ by the existence of mirrors and their positions.

The tiling with these tiles produces patterns with plane groups *p*6, *p*3, *p*3*m*1 and *p*31*m*, respectively (Fig. 21).

4. Conclusion

The *Hasba* is an empirical method where strict rules are applied at the start of the construction of a pattern. This method has frequently been adapted to carving and painting on wood. The concept of symmetry is ubiquitous in this

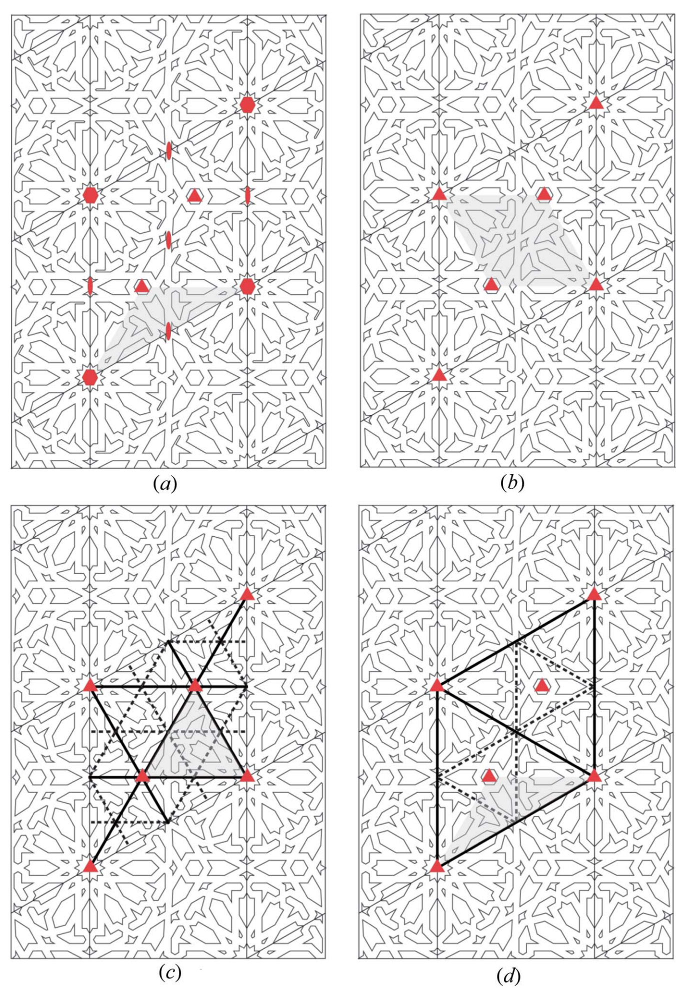


Figure 21
(*a*) The pattern with the plane group *p*6. (*b*) The pattern with the plane group *p*3. (*c*) The pattern with the plane group *p*3*m*1. (*d*) The pattern with the plane group *p*31*m*.

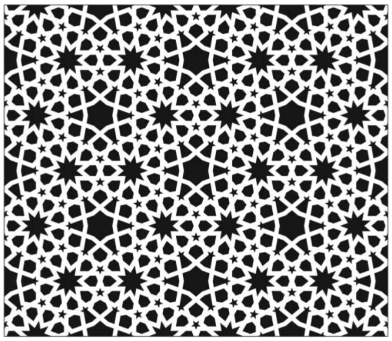


Figure 22
Ninefold pattern.

method. Patterns are underlaid by grids with a unit measure. Even hidden in the patterns, interlaced ribbons are naturally present in the designs; their thickness is constant and equal to unit division q. From a craftsmen's point of view, the ribbons constitute a way of checking the artistic validity of the patterns. Ribbons are infinitely continuous within the pattern as well as in repeat pattern. This is the main characteristic of the *Hasba*.

The *Hasba* method makes it possible to obtain patterns with n-fold rosettes. Both the complexity of the pattern and the symmetry of the rosettes increase with the value of the *Hasba*. The rosettes are considered as the basic element from which we generate the 17 symmetry groups.

Although in this article we have constructed the 17 groups with only ten- and 12-fold rosettes, other n-fold rosettes can be used to build a large number of patterns. Fig. 22 shows a $p6mm$ pattern generated from a ninefold rosette. As each rosette has its own symmetry, it is difficult to get all of the 17 symmetry groups with the same element.

Furthermore, all the generated patterns are highly charged with symmetry elements. The rosettes are hypersymmetric in comparison with the requirements of the plane groups they are in. Slabs of the structure with rosettes may have local mirror planes, which are not valid for the entire structure. The hypersymmetry may create confusion for artisans working on *zellige* (ceramic mosaics) when they build their patterns. However, this problem never occurs among artisans working on wood, because they begin to draw their designs on paper and cut a stencil, considered as the basic cell, before building patterns.

In our construction we have scrupulously respected the rules used by Moroccan craftsmen, except that we replaced the ruler and compass by drawing software (*Inkscape*; http://inkscape.org). Our patterns were presented to renowned master craftsmen in Moroccan geometric art, such as J. Benatia (a professor of traditional arts and calligraphy), S. Al Ouasrani and A. Boughali (master craftsmen at the workshop 'Traditional Moroccan Arts' in Marrakech). They validated the new patterns presented in the article, in terms of both construction and aesthetics.

At the practical level, the approach we have proposed, which combines both the traditional method of constructing patterns and the concept of the symmetry group, would offer to craftsmen new creative horizons to develop this ancestral art. It is true that in Morocco the ancient skills are alive and flourishing but no real major innovation can be observed.

At the academic level, this would be an opportunity for students to familiarize themselves with the plane groups, to better understand and use symmetry groups in crystallography and solid-state science. Some patterns introduced herein, although they are very close, would be more instructive for students and young researchers in their investigative work of symmetry groups. With various n-fold rosettes, we can generate very different groups (Fig. 22).

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