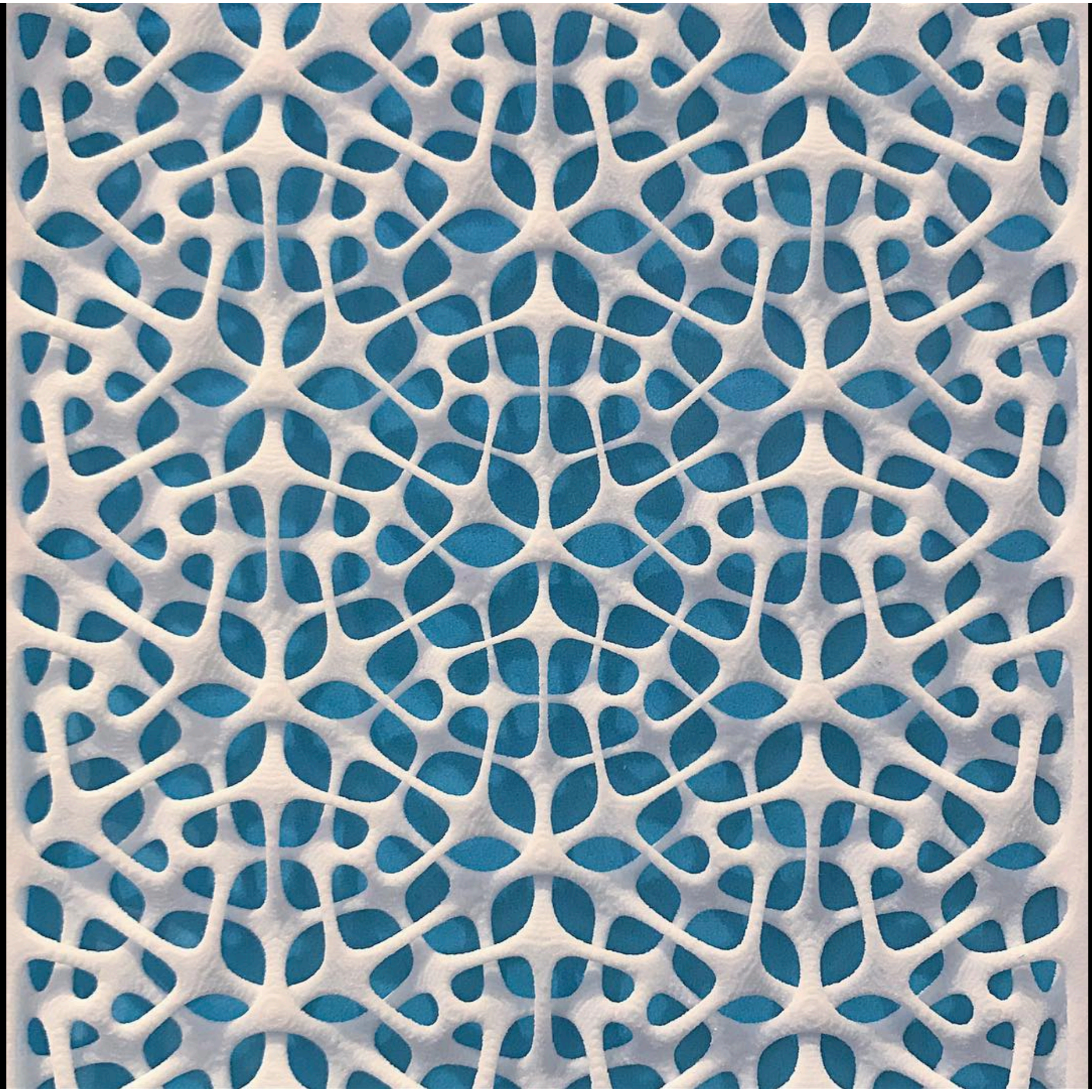
Computational Fabrication

CS 491 and 591
Professor: Leah Buechley
https://handandmachine.cs.unm.edu/classes/Computational_Fabrication_Spring2021/

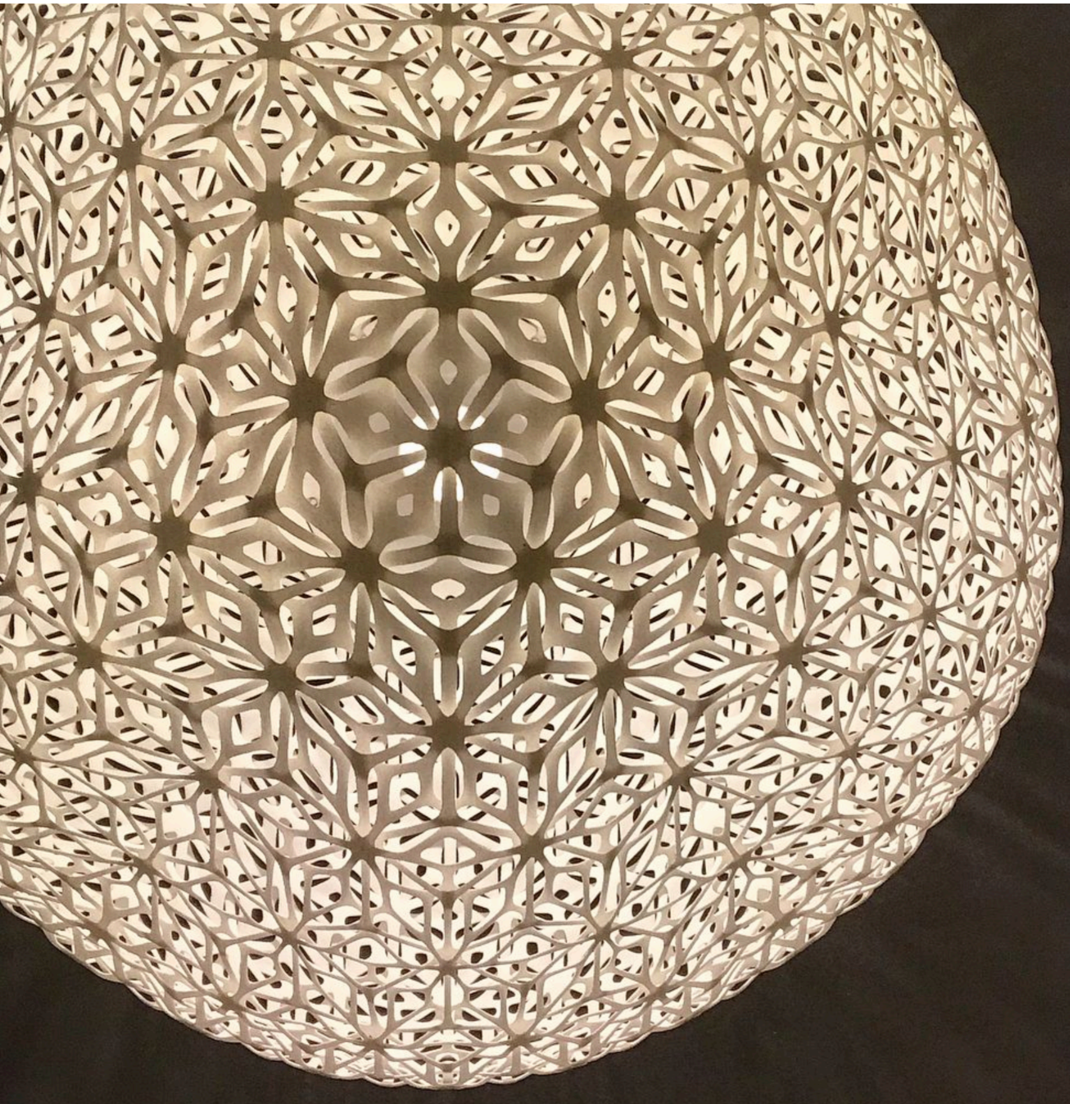
Artist: Travis Fitch

https://fitchwork.com/
https://www.instagram.com/fitchwork/
https://www.futurecurrent.net/travis-fitch


Travis Fitch



Travis Fitch



Travis Fitch

Travis Fitch

Class Schedule Check In

Week 11, October 28: Tiling

Tuesday	Introduction to Tiling Categories of tiles and tilings Escher Tile Design Website	
Thursday	Bravais lattices and periodic tilings Constructing tiles and tilings	**Small Assignment: Final Project Proposals**

Week 12, November 4: Tiling cont.

Tuesday	Tiling cont. Tiling non-planar surfaces Surface morph	
Thursday	Guest lecture: Scott Hudson	**Small Assignment: Scott Hudson research**

Week 13, November 11

Tuesday		**Large Assignment 5: Tiling**

Tiling

Huge topic! We’ll scratch the surface a little.



2D Tiling/Tessellations

What is a Tiling?

A **tiling** (of the plane) is a collection of **tiles** (subsets of the plane), which cover the plane without gaps or overlaps. We also require that each tile consists of a single connected piece without holes or lines.

http://pi.math.cornell.edu/~mec/2008-2009/KathrynLindsey/PROJECT/Page1.htm

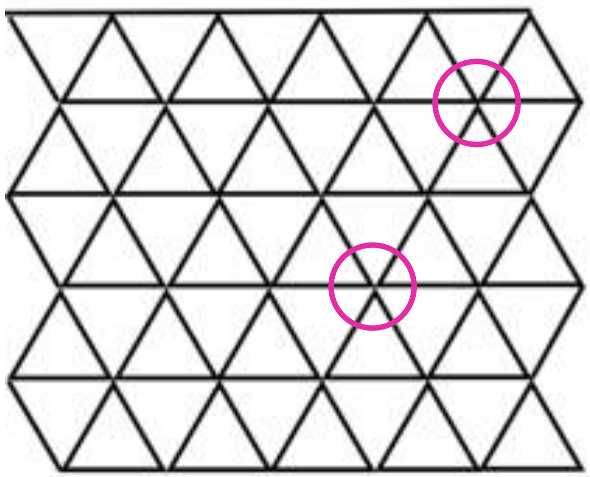
Regular Tilings

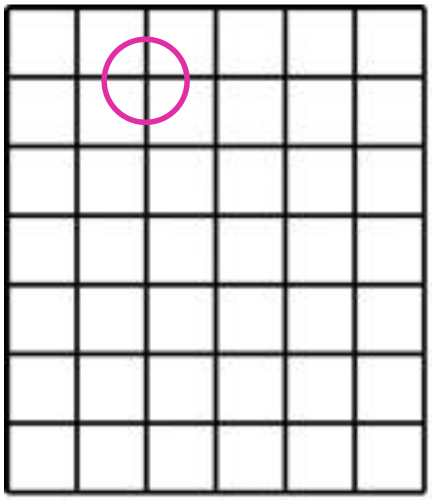
Tiling by a single regular polygon

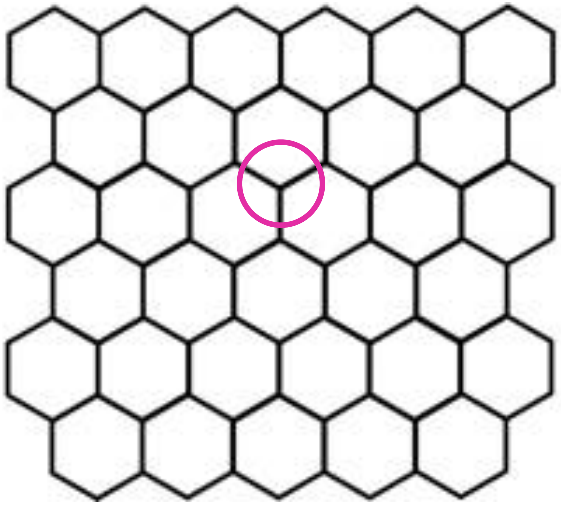
Regular polygon: shapes where all sides and angles are the same

Regular tiling: all vertices are the same

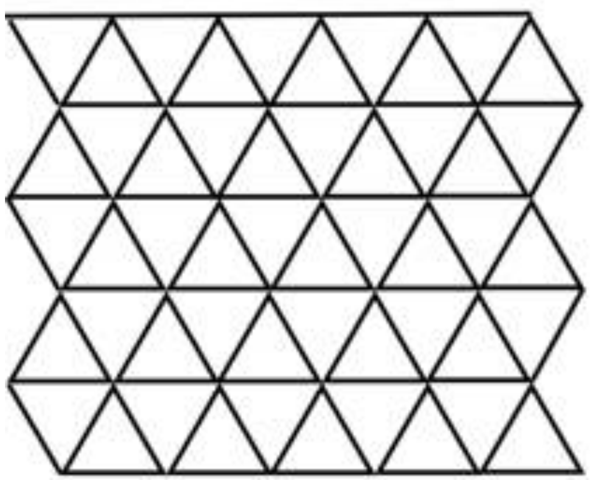
3 Regular Tilings

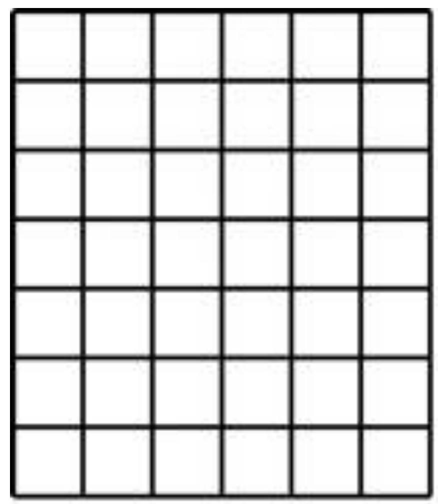


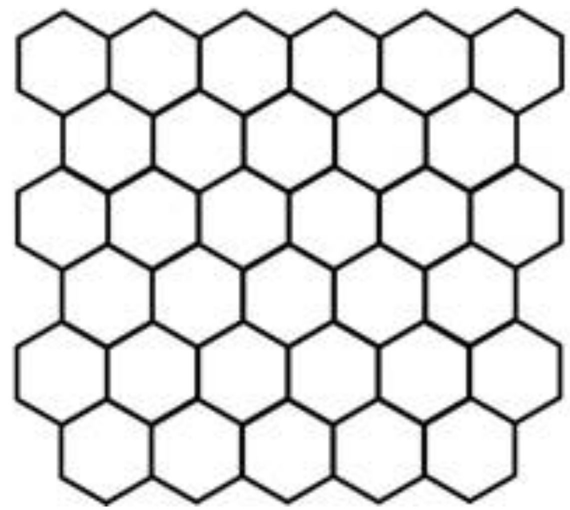




The Only Regular Tilings!

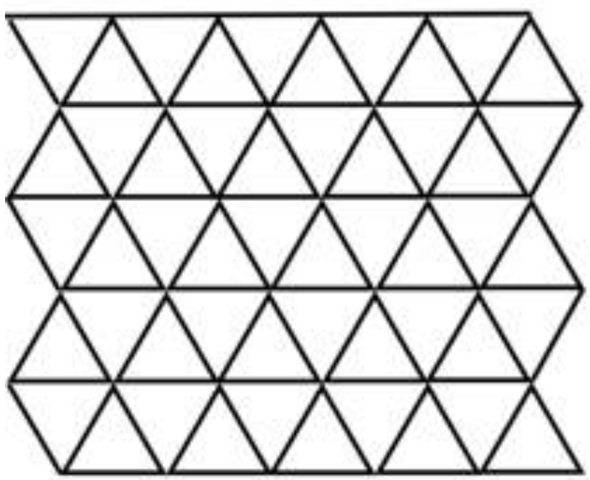




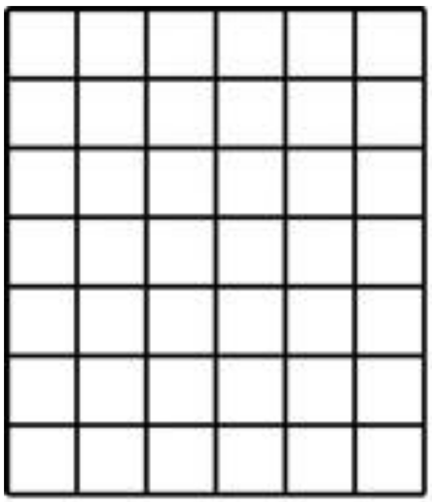


Why?

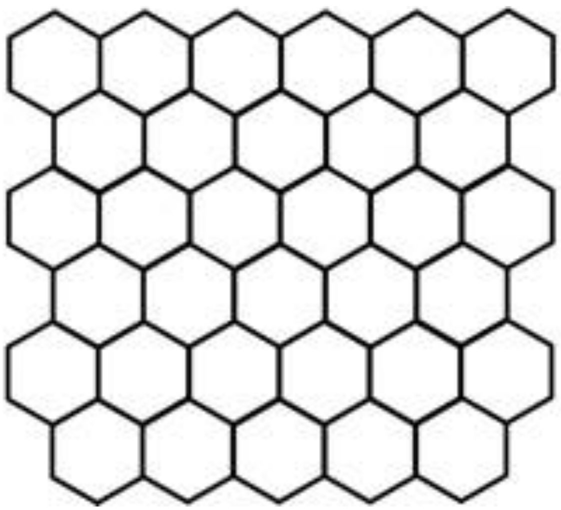
interior angle x integer = 360



angle = 60
60 x 6 = 360



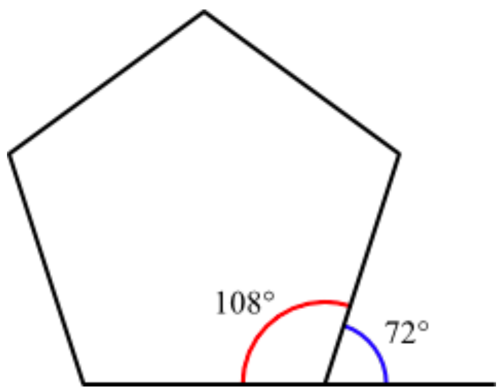
angle = 90
90 x 4 = 360



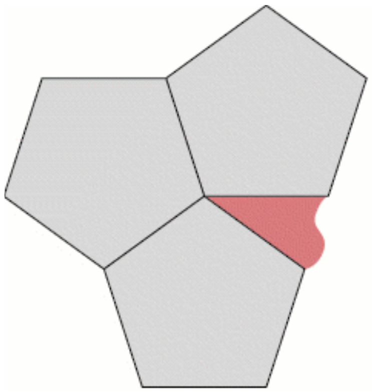
angle = 120
120 x 3 = 360

Why not Pentagons?

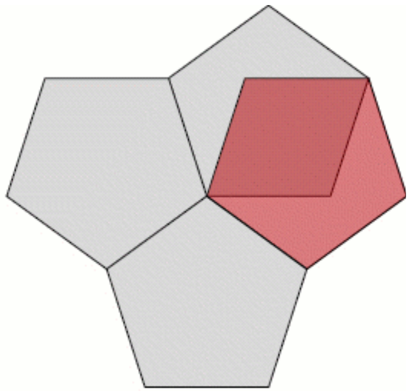
interior angle x integer = 360



pentagon
interior angle = 108

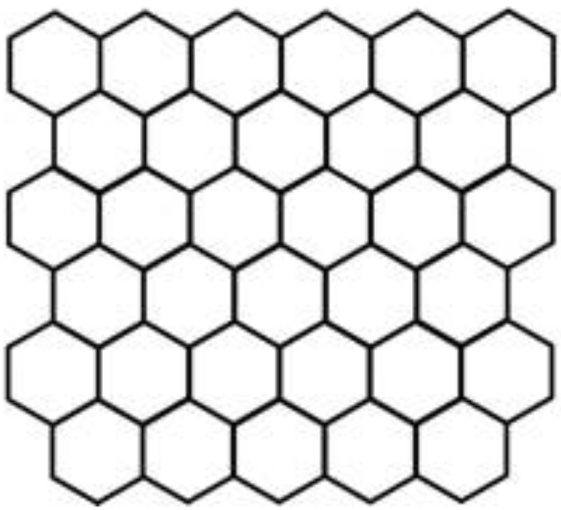


108 x 3 = 324

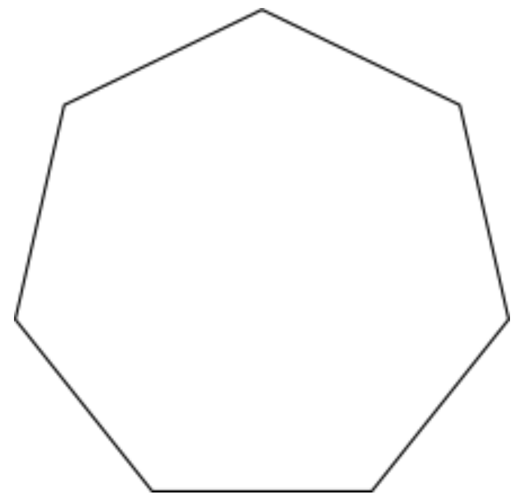


108 x 4 = 432

Why not greater than 6 sides?

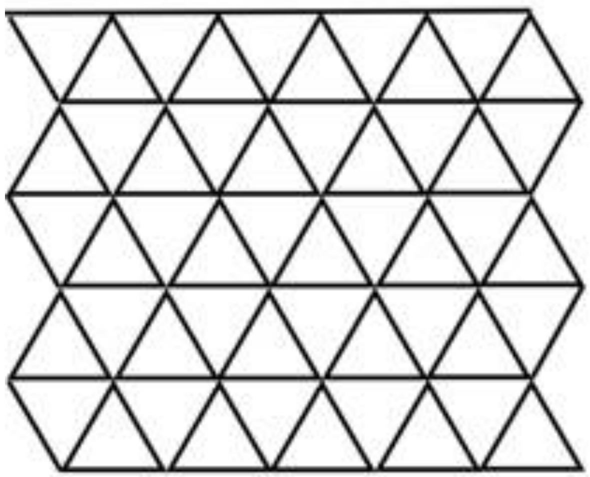


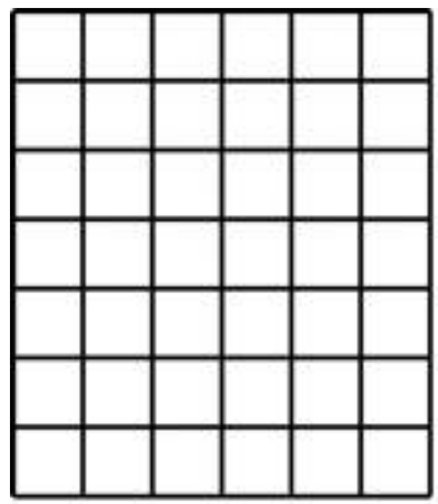
angle = 120
120 x 3 = 360

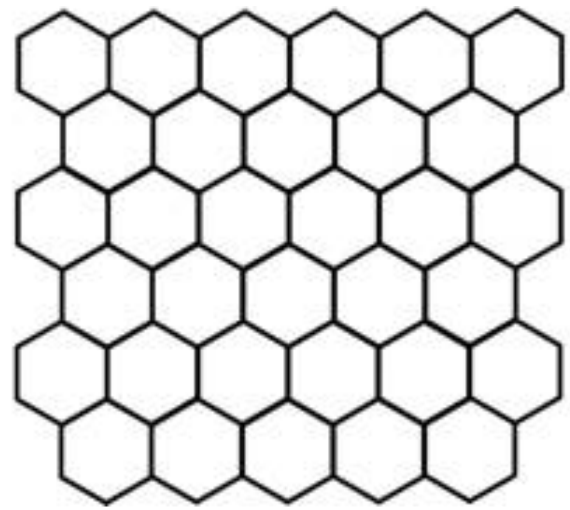


heptagon
angle = 128
128 x 3 = 384

There are Only 3 Regular Tilings





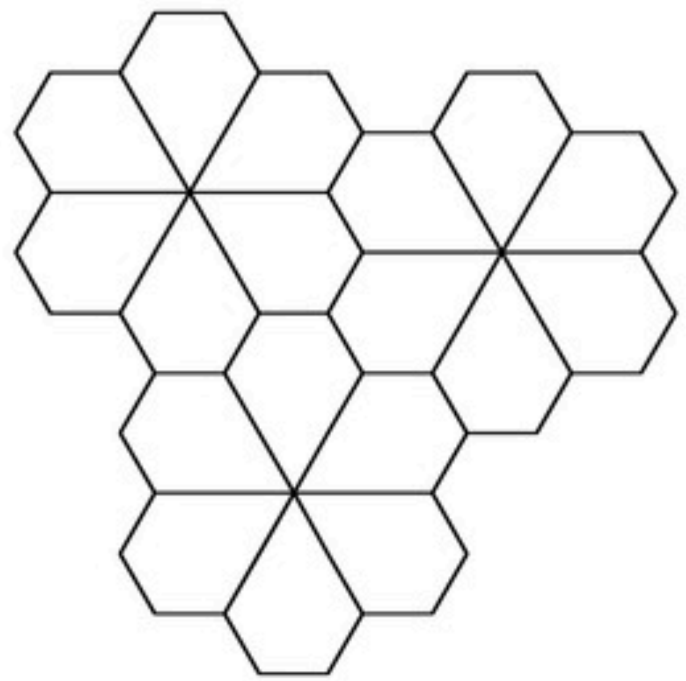


Monohedral Tilings

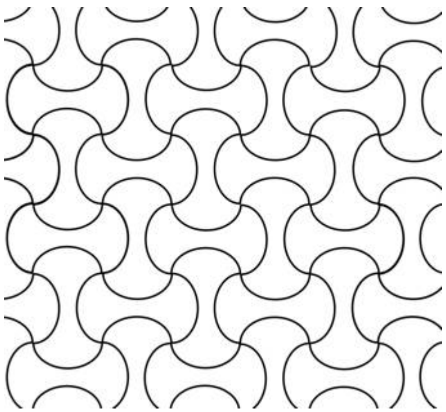
Tiling by a single shape

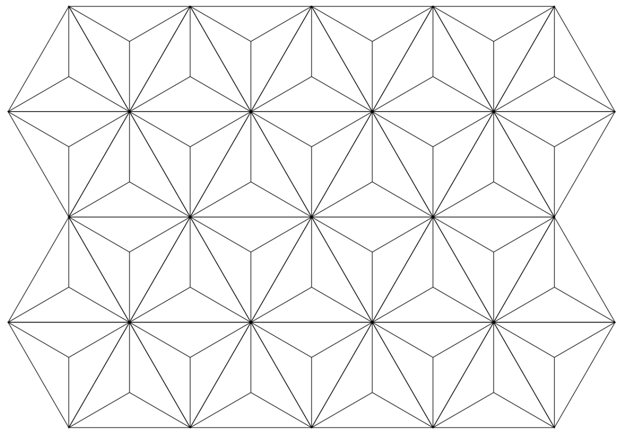
No other constraints

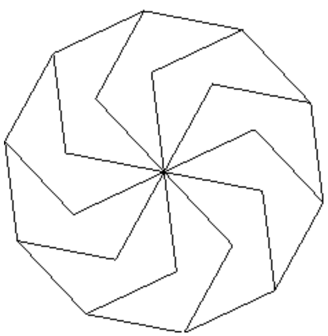
Example: a tiling with non-regular pentagons

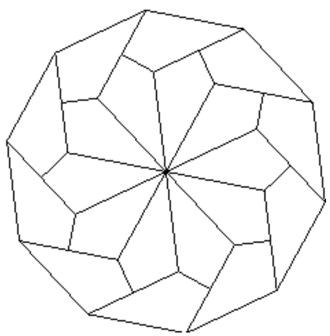


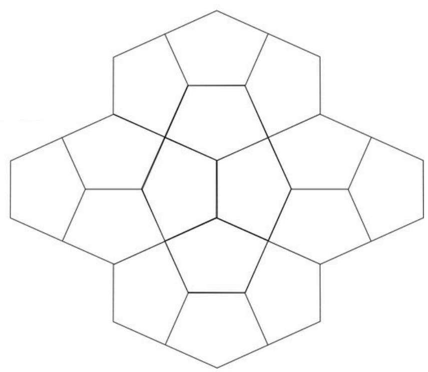
Lots of Monohedral Tilings!

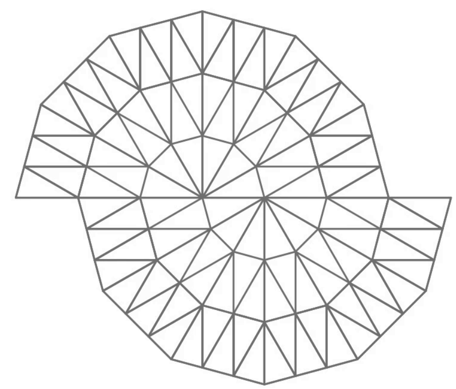






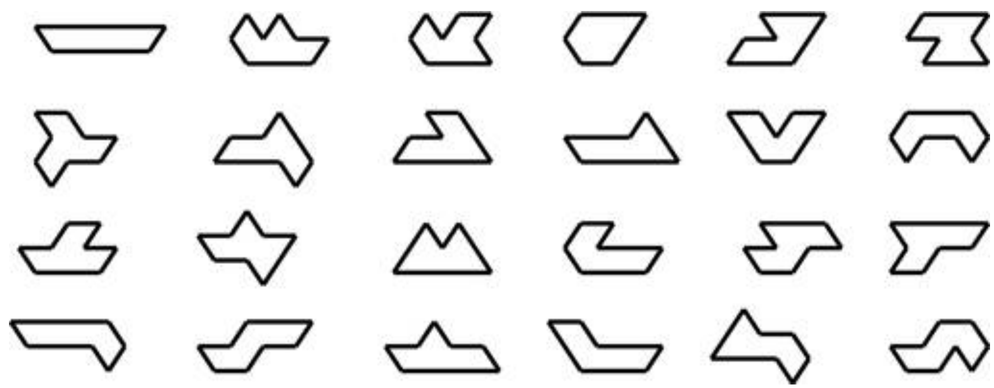






Monohedral Tilings: a Question

If you are given a tile, can you determine if it tiles the plane?

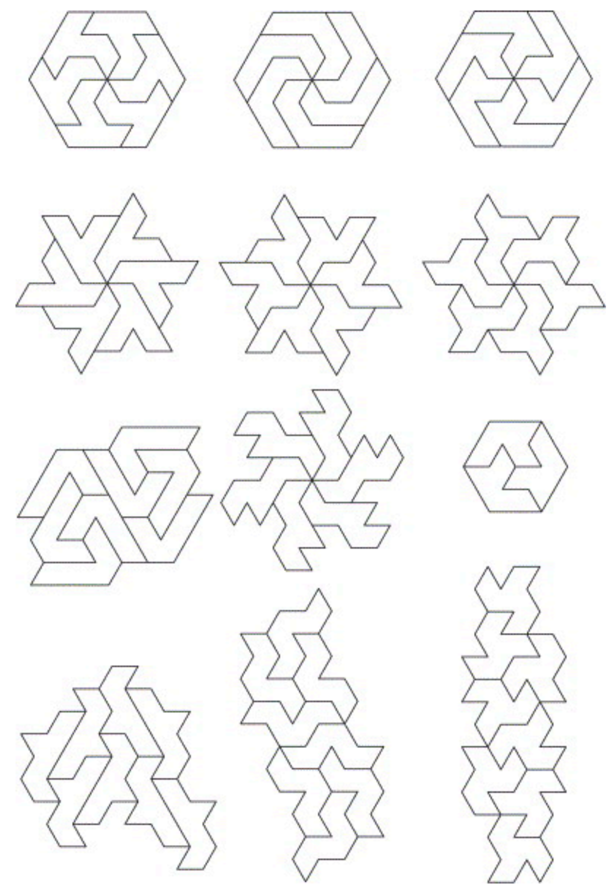


Monohedral Tilings: a Question

If you are given a tile, can you determine if it tiles the plane?

An open question!

May be undecidable. We don't know!



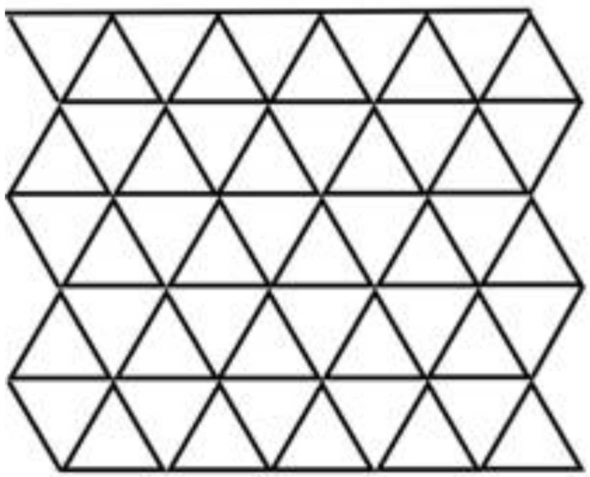
http://www.ams.org/notices/201003/rtx100300343p.pdf

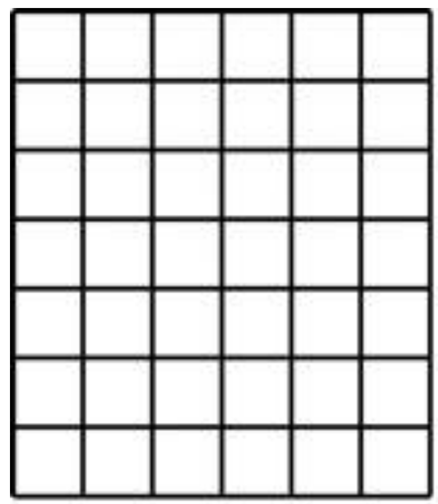
http://math.tsukuba.ac.jp/ant/Sympo/GS_kyoto1.pdf

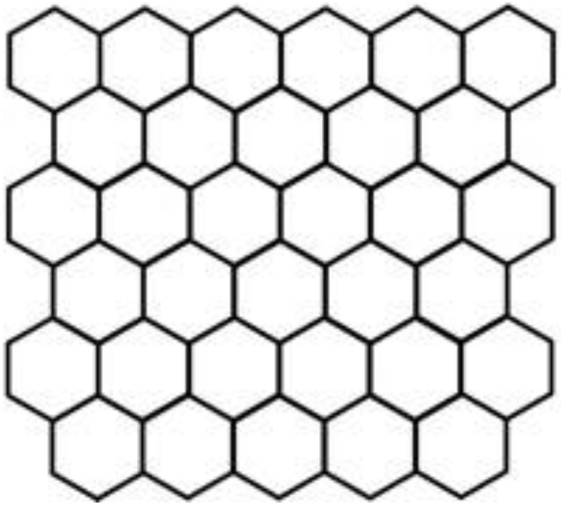
http://www.cs.bc.edu/~straubin/cs385-07/tiling

Lots of interesting open tiling questions in CS theory!

Back to Regular Tilings





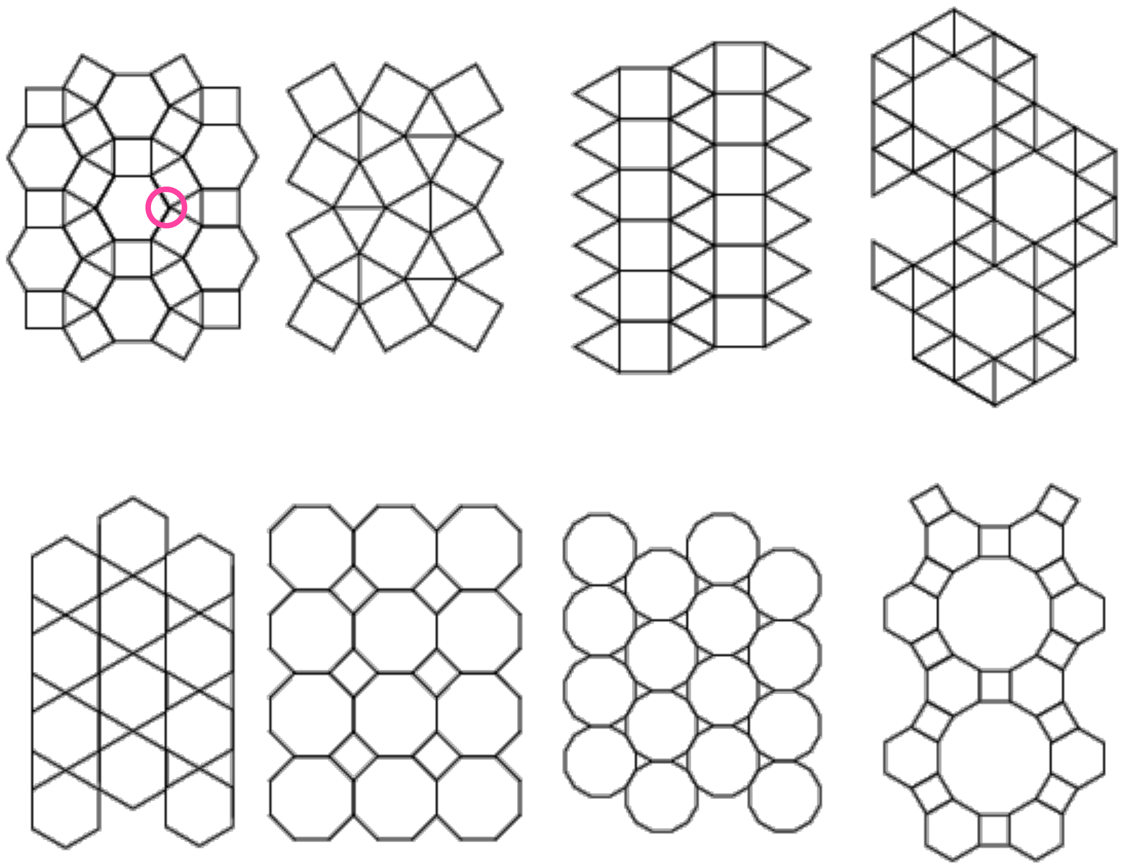


Semi-Regular Tilings

Tilings by one or more regular polygons

All **vertices** are the same

Eight Semi-Regular Tilings

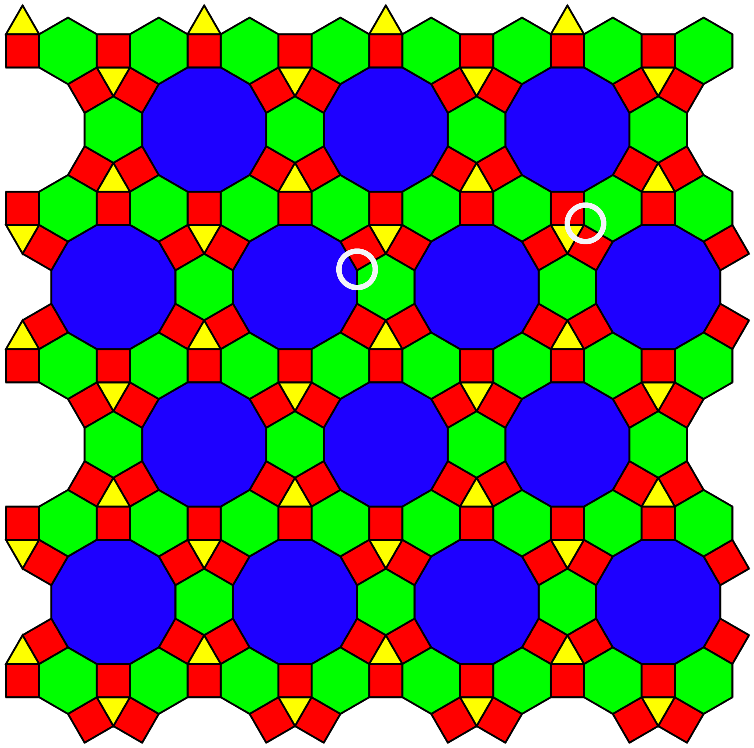


Demi-Regular Tilings

Also known as 2-Uniform Tilings

Tilings by one or more regular polygons

Two types of vertices

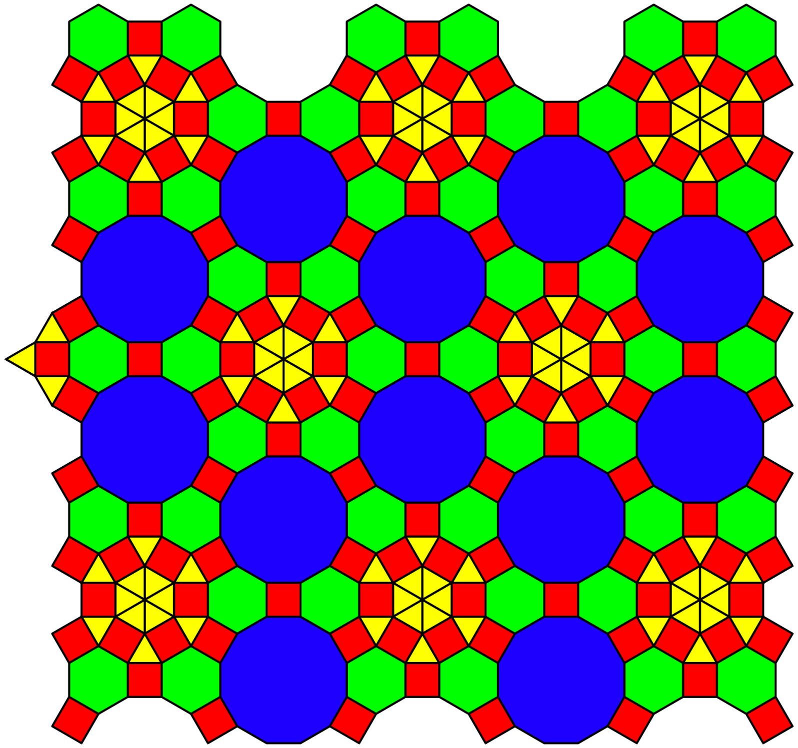


k-Uniform Tilings

Tilings by one or more regular polygons

k types of vertices

Example: 5-uniform tiling



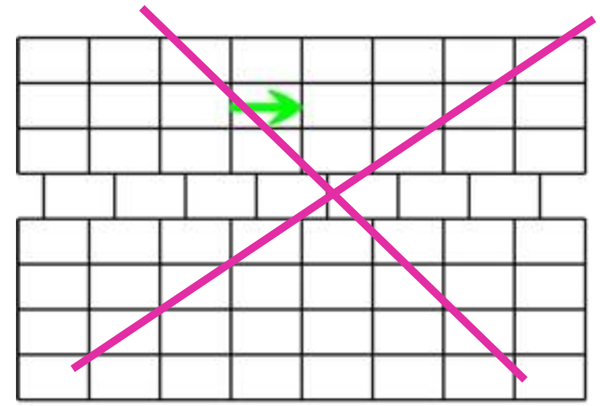
https://en.wikipedia.org/wiki/List_of_k-uniform_tilings

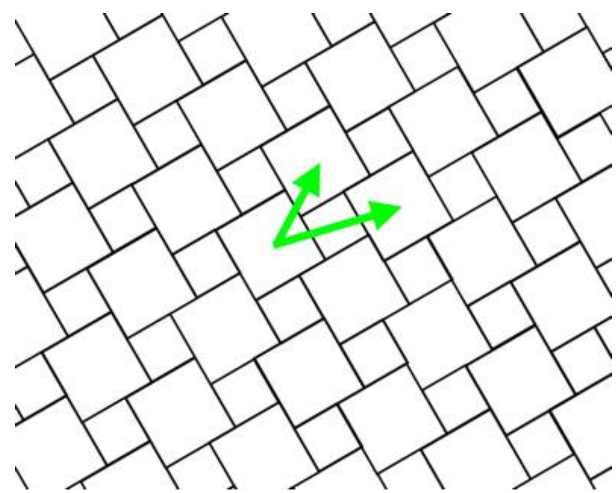
Different Kinds of Tilings

Periodic Tilings

A tiling that you can replicate **by translation** in at least two non-parallel directions.

Think about wallpaper. A tiling you can create a wallpaper from.





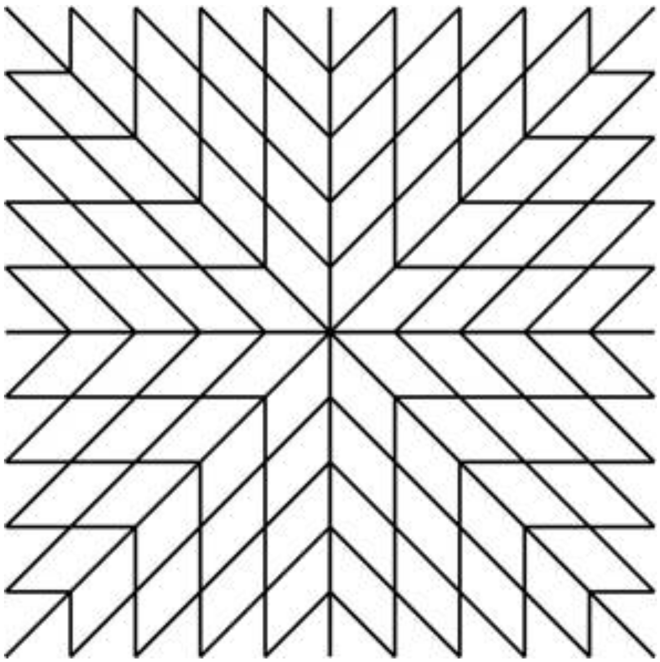
http://pi.math.cornell.edu/~mec/2008-2009/KathrynLindsey/PROJECT/Page4.htm

Nonperiodic Tilings

A tiling that you cannot replicate **by translation**

Think about wallpaper. A tiling you cannot create a wallpaper from.

Note: does not rule out radial symmetry

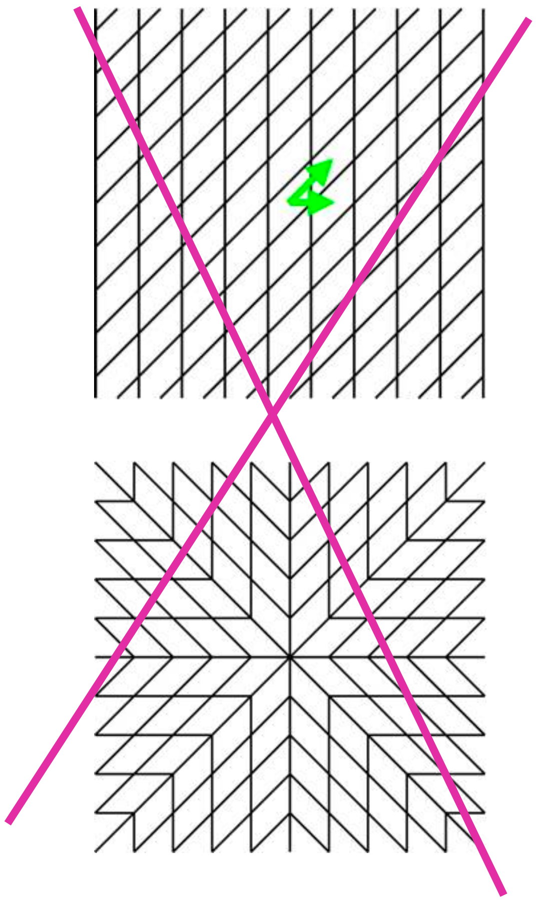


http://pi.math.cornell.edu/~mec/2008-2009/KathrynLindsey/PROJECT/Page4.htm

Aperiodic Tilings

A set of tiles that can **only** create Non-periodic tilings.

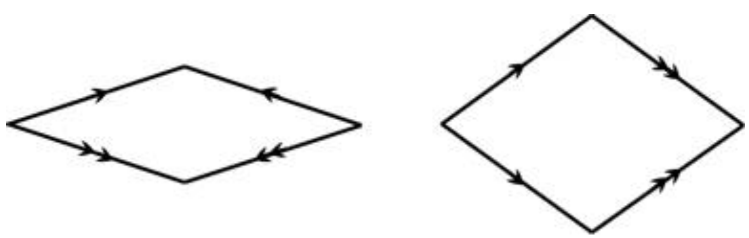
Negative example on the right.

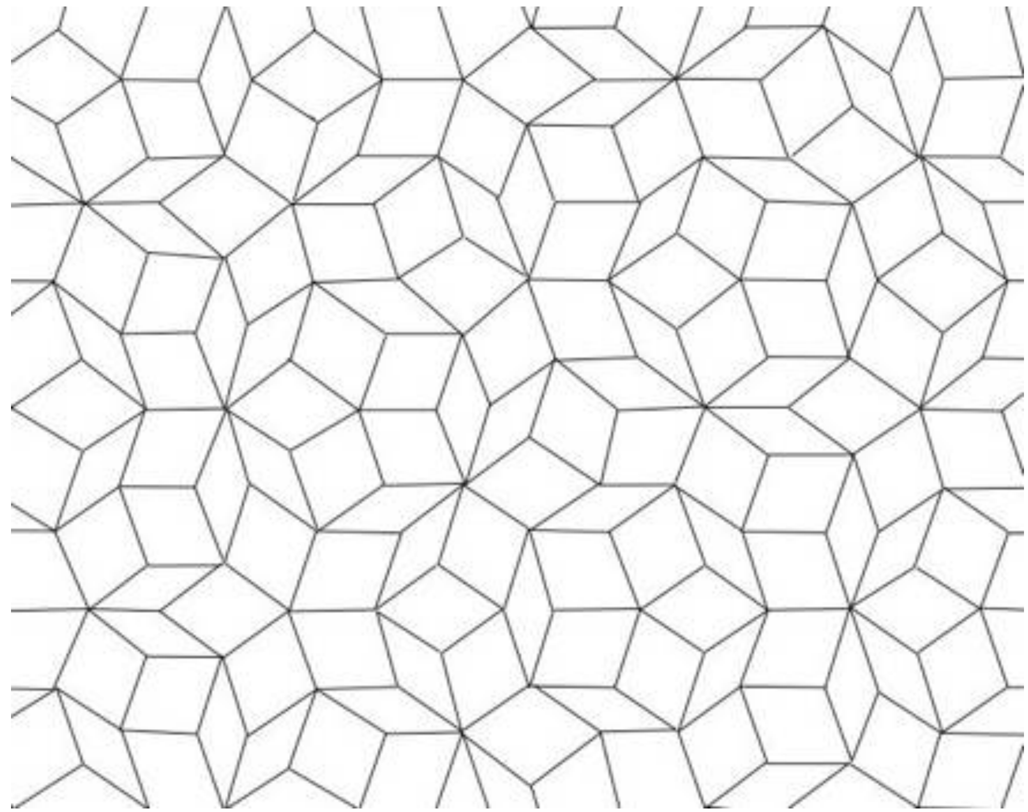


http://pi.math.cornell.edu/~mec/2008-2009/KathrynLindsey/PROJECT/Page5.htm

Aperiodic Tiling: Penrose Tiling

Tiles





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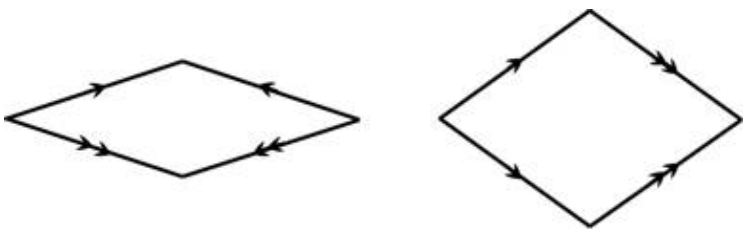
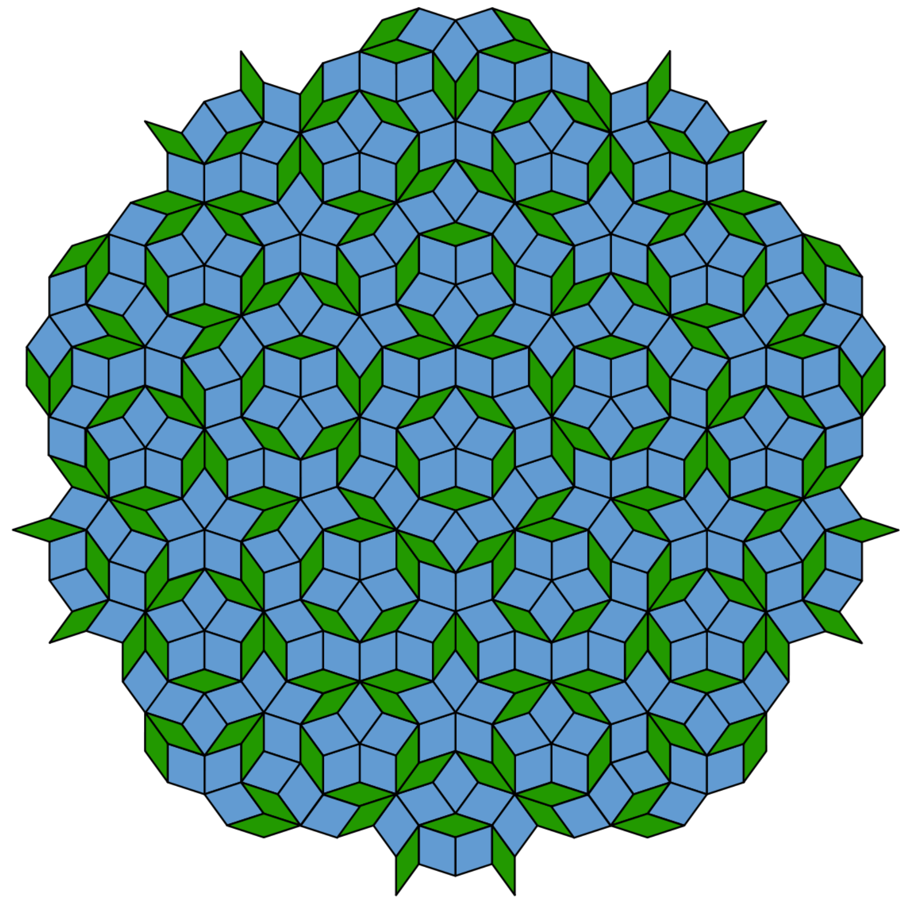
Aperiodic Tiling: Penrose Tiling

Tiles





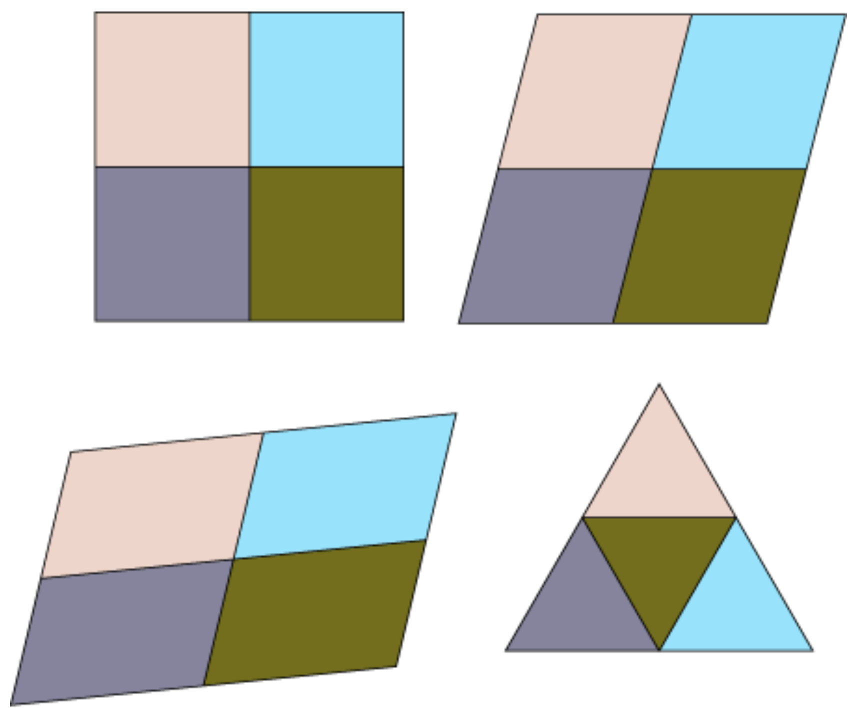
Note: does not rule out radial symmetry

http://pi.math.cornell.edu/~mec/2008-2009/KathrynLindsey/PROJECT/Page5.htm

Rep Tiles

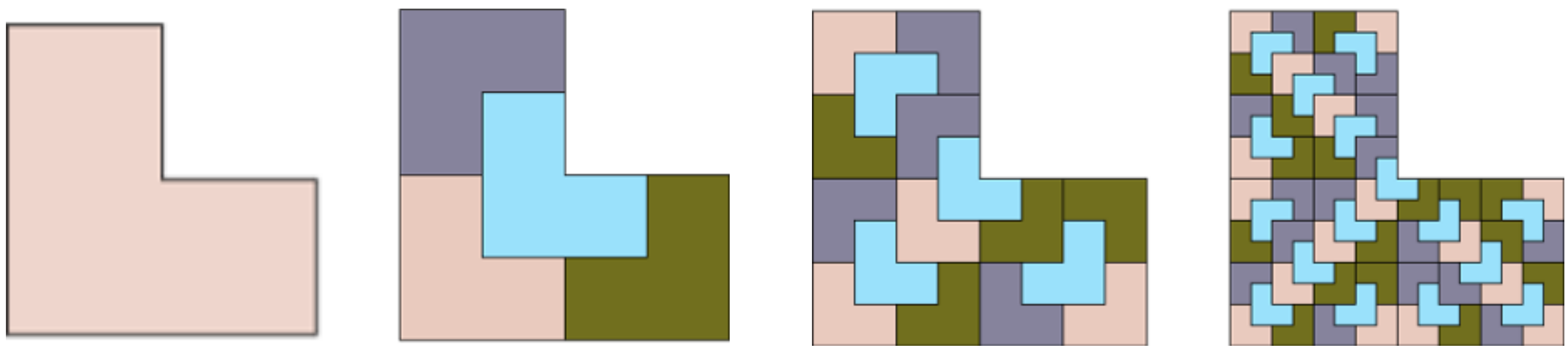
Self-Similar/Fractal Tiles

Rep-Tiles



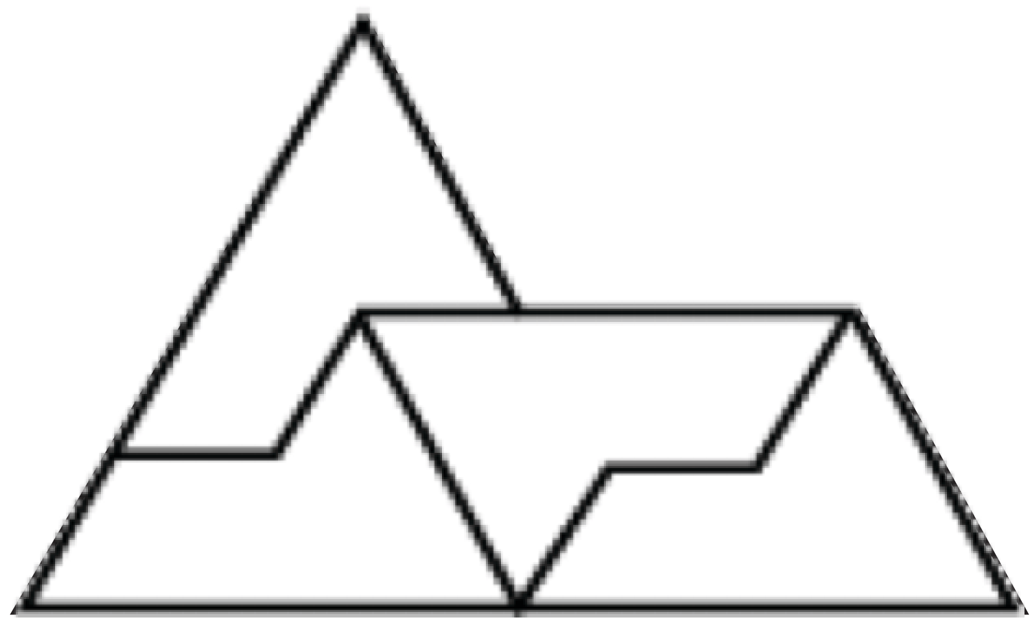
Rep-Tiles

Can you break the shape into 4 copies of itself?



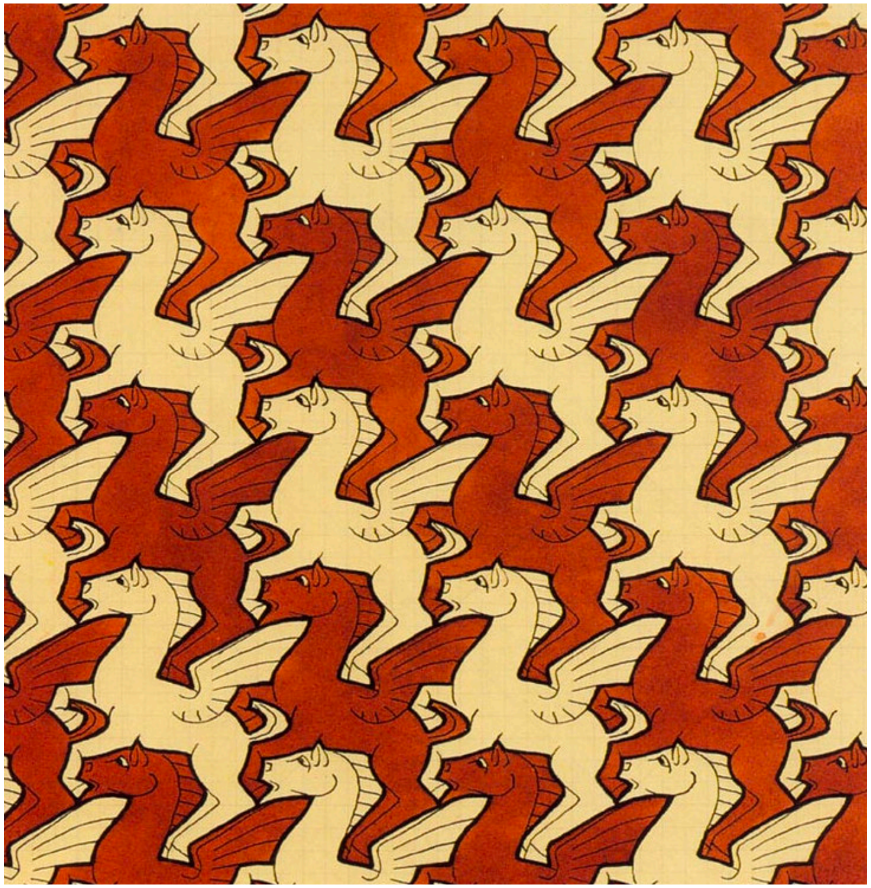
This one?

This one?



Escher Tiles

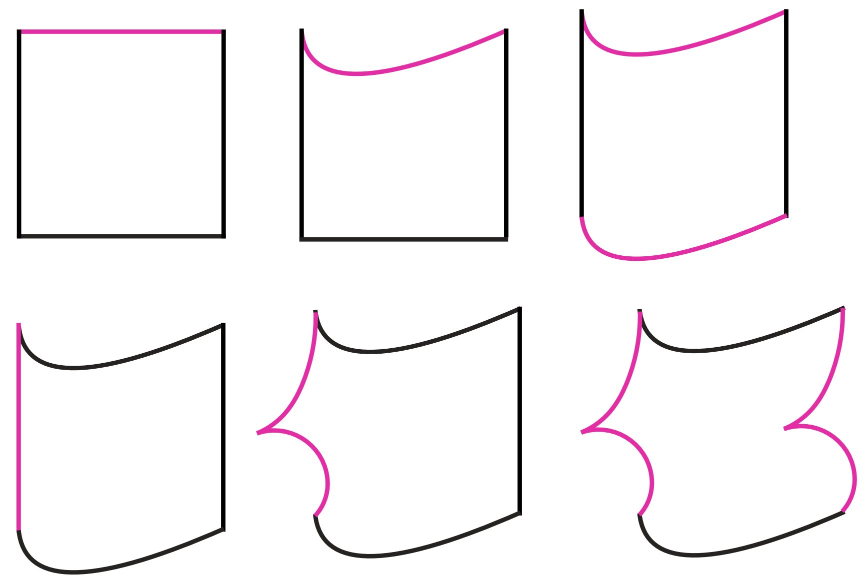
M.C. Escher



Creating Interesting Tiles

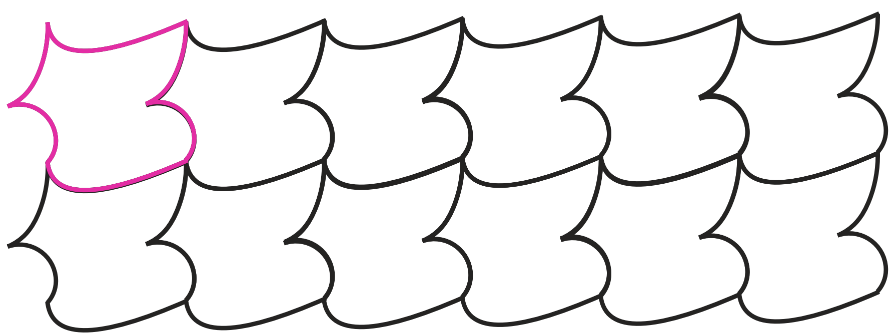
How to create your own tiles using existing tilings as a starting point.

Modify two matching edges or vertices in the same way



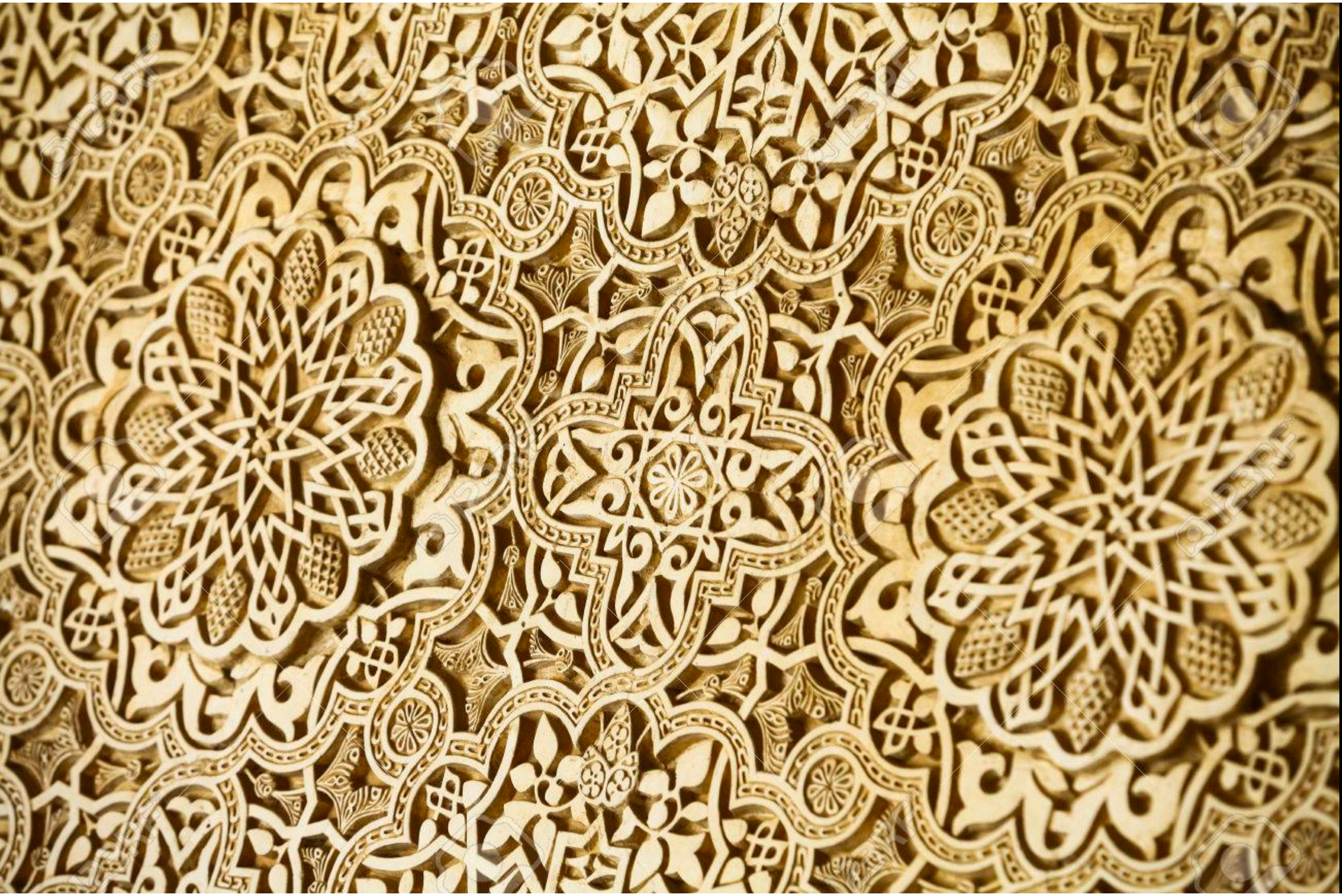
https://www.rochester.edu/pr/Review/V79N3/0603_schattschneider_side.html

Creating Interesting Tiles



https://tiled.art/en/create/?id=Quad1

2.5 D Tiling/Tessellations

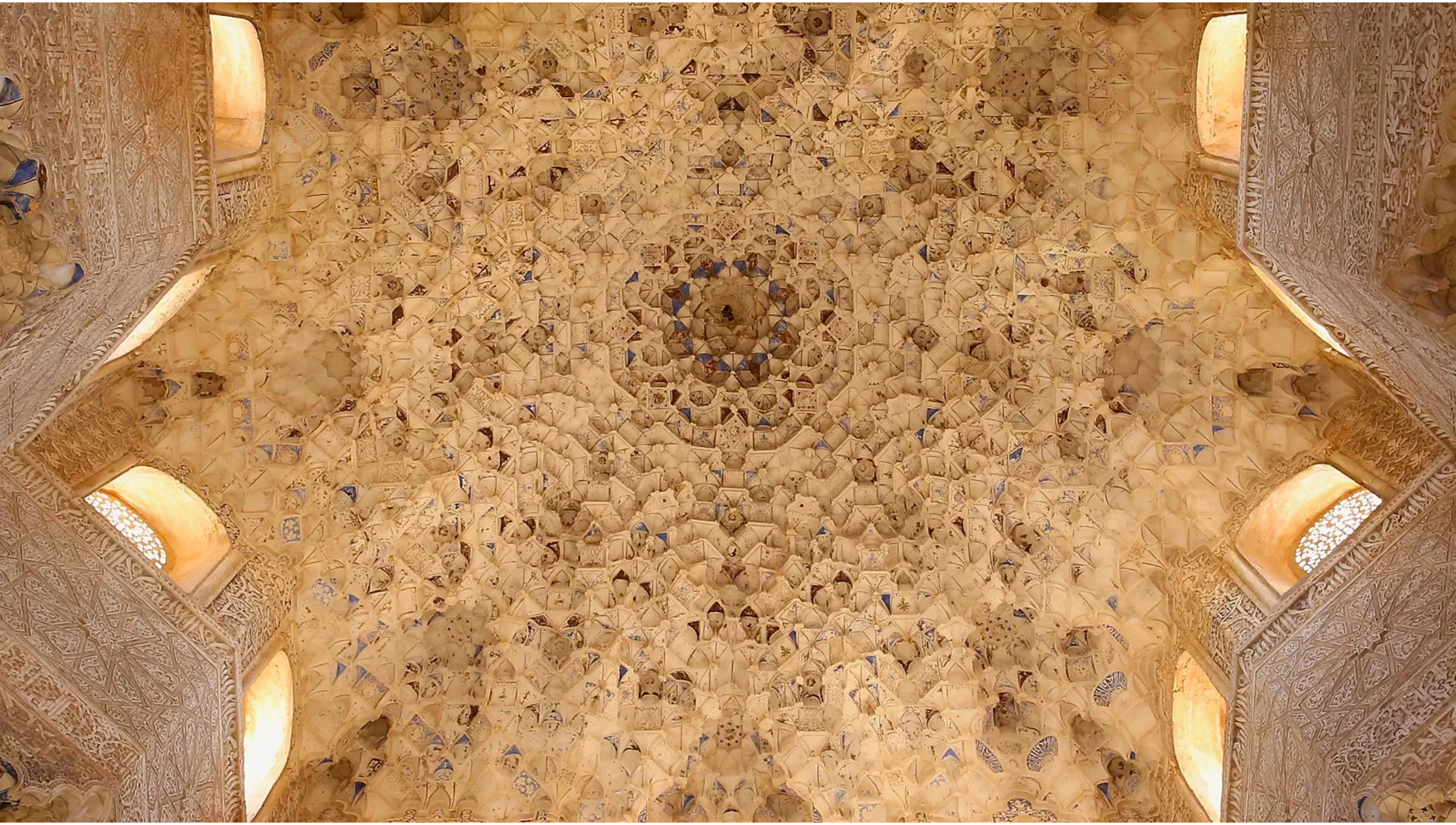


Alhambra Mosque

Alhambra Mosque

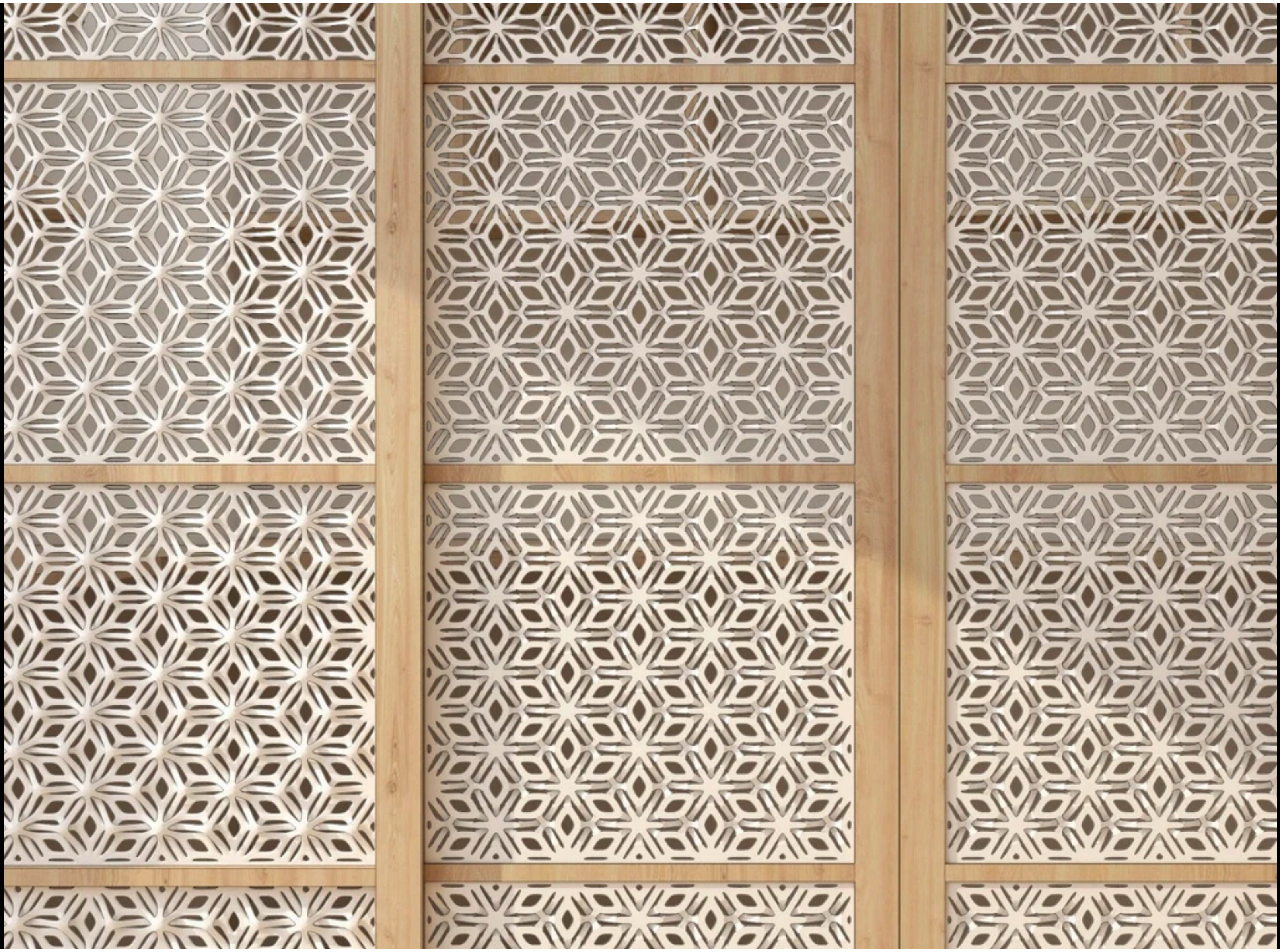
Alhambra Mosque

Alhambra Mosque



Raffello Galiotto for Lithos Design
https://www.lithosdesign.com/


Travis Fitch



Creating Interesting Tiles

Use one of the foundational tilings as a starting point.

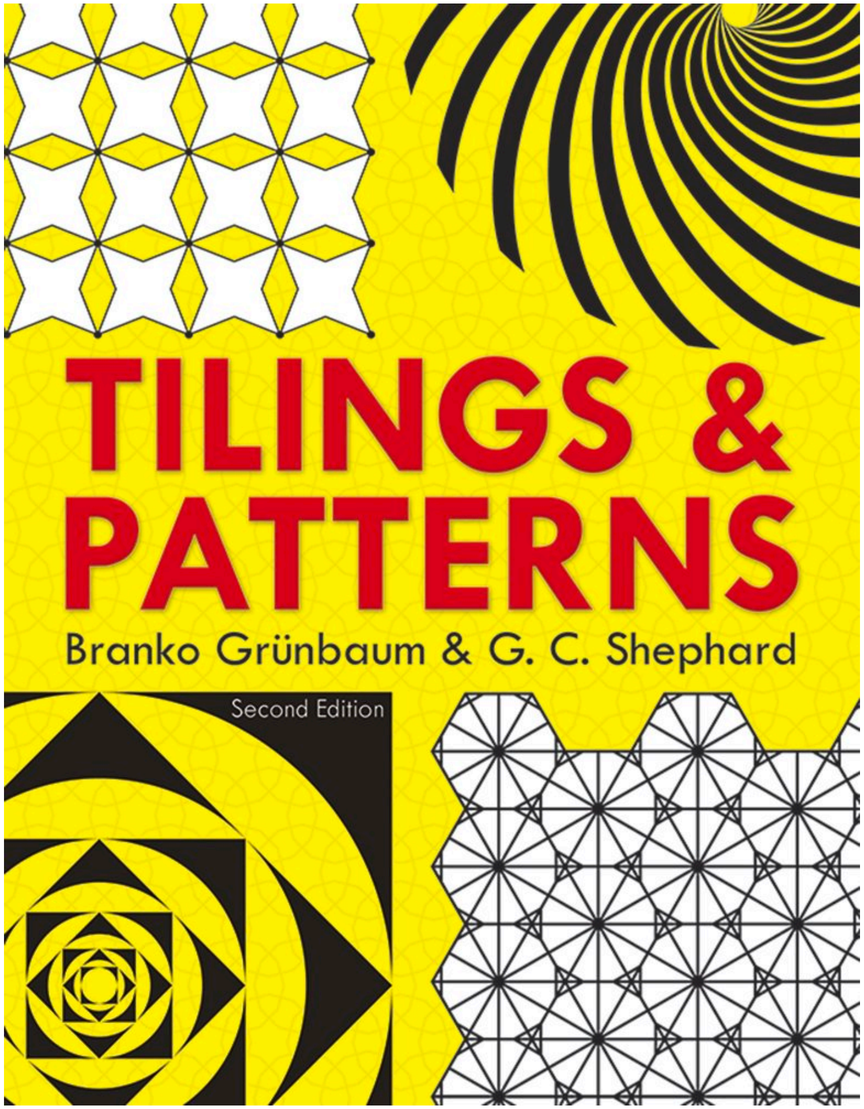
Add complexity (in 2D or 3D). Constraint: maintain edge relationships

Tile through repetition, consider fractalization

Morph across surface

questions?

Categorizations of Tiles

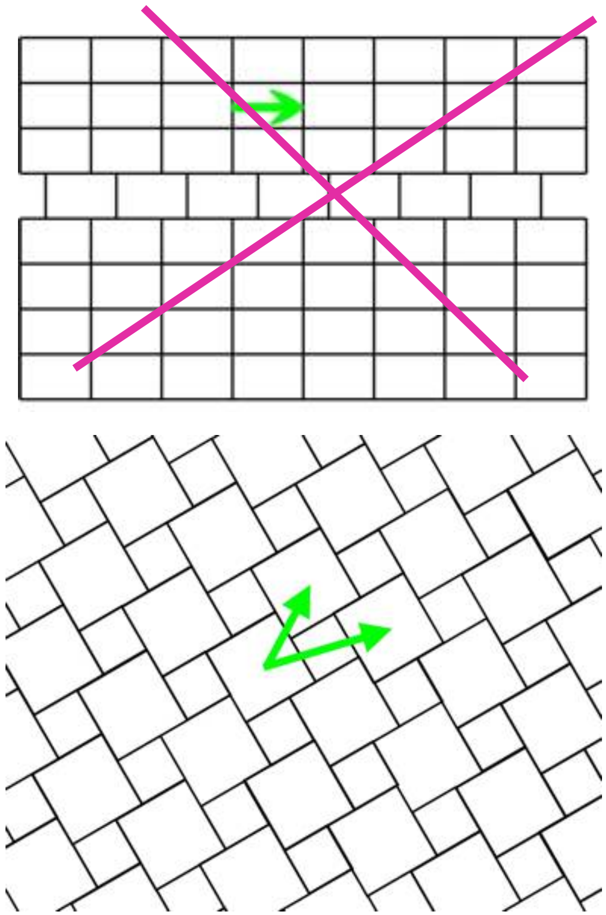


(Periodic) Tile Generation

Periodic Tilings

A tiling that you can replicate **by translation** in at least two non-parallel directions.

Think about wallpaper. A tiling you can create a wallpaper from.



http://pi.math.cornell.edu/~mec/2008-2009/KathrynLindsey/PROJECT/Page4.htm

Periodic Tilings and Wallpaper Groups

- Any periodic tiling can be characterized as a "wallpaper".
- Wallpaper Groups: formal categories that describe the types of symmetries present in a tiling
- Describing symmetry = describing transformations (translation, rotation, reflection). Useful information for constructing tilings.

https://en.wikipedia.org/wiki/Wallpaper_group
https://mathworld.wolfram.com/WallpaperGroups.html

17 Wallpaper Groups (2D)

Square
$[4,4]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$		Monotropic
p2 (2222) $p\bar{2}$	$[4,1^+,4]^+$ $[1^+,4,4,1^+]^+$	Ditropic
pgg (22×) p_g2_g	$[4^+,4^+]$	Diglide
pmm (*2222) p2	$[4,1^+,4]$ $[1^+,4,4,1^+]$	Discopic
cmm (2*22) c2	$[(4,4,2^+)]$	Dirhombic
p4 (442) $p\bar{4}$	$[4,4]^+$	Tetratropic
p4g (4*2) p_g4	$[4^+,4]$	Tetragyro
p4m (*442) p4	$[4,4]$	Tetrascopic

Rectangular
$[\infty_h,2,\infty_v]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$	$[\infty^+,2,\infty^+]$	Monotropic
p2 (2222) $p\bar{2}$	$[\infty,2,\infty]^+$	Ditropic
pg(h) (××) p_g1	h: $[\infty^+,(2,\infty)^+]$	Monoglide
pg(v) (××) p_g1	v: $[(\infty,2)^+,\infty^+]$	Monoglide
pgm (22*) p_g2	h: $[(\infty,2)^+,\infty]$	Digyro
pmg (22*) p_g2	v: $[\infty,(2,\infty)^+]$	Digyro
pm(h) (**) p1	h: $[\infty^+,2,\infty]$	Monoscopic
pm(v) (**) p1	v: $[\infty,2,\infty^+]$	Monoscopic
pmm (*2222) p2	$[\infty,2,\infty]$	Discopic

Rhombic
$[\infty_h,2^+,\infty_v]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$	$[\infty^+,2^+,\infty^+]$	Monotropic
p2 (2222) $p\bar{2}$	$[\infty,2^+,\infty]^+$	Ditropic
cm(h) (*×) c1	h: $[\infty^+,2^+,\infty]$	Monorhombic
cm(v) (*×) c1	v: $[\infty,2^+,\infty^+]$	Monorhombic
pgg (22×) p_g2_g	$[((\infty,2)^+)^{[2]}]$	Diglide
cmm (2*22) c2	$[\infty,2^+,\infty]$	Dirhombic

Parallelogrammatic (oblique)

p1 (°) $p\bar{1}$	Monotropic
p2 (2222) $p\bar{2}$	Ditropic

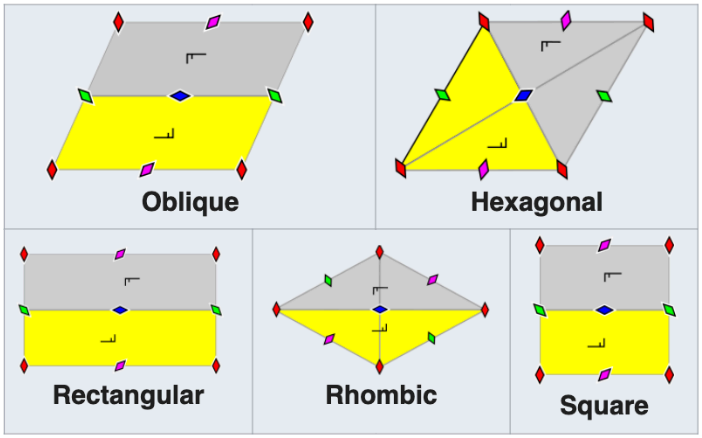
Hexagonal/Triangular
$[6,3]$ / $[3^{[3]}]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$		Monotropic
p2 (2222) $p\bar{2}$	$[6,3]^{\Delta}$	Ditropic
cmm (2*22) c2	$[6,3]^{⅄}$	Dirhombic
p3 (333) $p\bar{3}$	$[1^+,6,3^+]$ $[3^{[3]}]^+$	Tritropic
p3m1 (*333) p3	$[1^+,6,3]$ $[3^{[3]}]$	Triscopic
p31m (3*3) h3	$[6,3^+]$	Trigyro
p6 (632) $p\bar{6}$	$[6,3]^+$	Hexatropic
p6m (*632) p6	$[6,3]$	Hexascopic

https://en.wikipedia.org/wiki/Wallpaper_group

Group *p2* (2222) [edit]

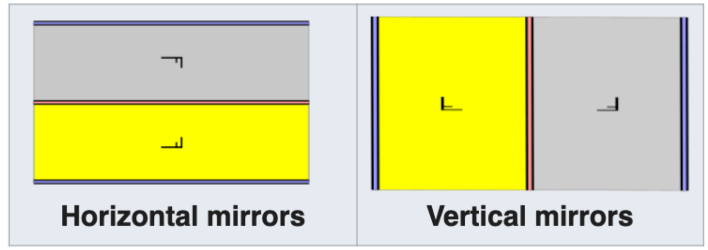
Cell structures for *p2* by lattice type



- Orbifold signature: 2222
- Coxeter notation (rectangular): $[\infty,2,\infty]^+$
- Lattice: oblique
- Point group: C_2
- The group ***p2*** contains four rotation centres of order two (180°), but no reflections or glide reflections.

Group *pm* (**) [edit]

Cell structure for *pm*



- Orbifold signature: **
- Coxeter notation: $[\infty,2,\infty^+]$ or $[\infty^+,2,\infty]$
- Lattice: rectangular
- Point group: D_1
- The group ***pm*** has no rotations. It has reflection axes, they are all parallel.

https://en.wikipedia.org/wiki/Wallpaper_group

17 Wallpaper Groups (2D)

https://en.wikipedia.org/wiki/Wallpaper_group

Square
[4,4]

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) p$\bar{1}$		Monotropic
p2 (2222) p$\bar{2}$	$[4,1^+,4]^+$ $[1^+,4,4,1^+]^+$	Ditropic
pgg (22×) p_g2_g	$[4^+,4^+]$	Diglide
pmm (*2222) p2	$[4,1^+,4]$ $[1^+,4,4,1^+]$	Discopic
cmm (2*22) c2	$[(4,4,2^+)]$	Dirhombic
p4 (442) p$\bar{4}$	$[4,4]^+$	Tetratropic
p4g (4*2) p_g4	$[4^+,4]$	Tetragyro
p4m (*442) p4	[4,4]	Tetrascopic

Rectangular
$[\infty_h,2,\infty_v]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) p$\bar{1}$	$[\infty^+,2,\infty^+]$	Monotropic
p2 (2222) p$\bar{2}$	$[\infty,2,\infty]^+$	Ditropic
pg(h) (××) p_g1	h: $[\infty^+,(2,\infty)^+]$	Monoglide
pg(v) (××) p_g1	v: $[(\infty,2)^+,\infty^+]$	Monoglide
pgm (22*) p_g2	h: $[(\infty,2)^+,\infty]$	Digyro
pmg (22*) p_g2	v: $[\infty,(2,\infty)^+]$	Digyro
pm(h) (**) p1	h: $[\infty^+,2,\infty]$	Monoscopic
pm(v) (**) p1	v: $[\infty,2,\infty^+]$	Monoscopic
pmm (*2222) p2	$[\infty,2,\infty]$	Discopic

Rhombic
$[\infty_h,2^+,\infty_v]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) p$\bar{1}$	$[\infty^+,2^+,\infty^+]$	Monotropic
p2 (2222) p$\bar{2}$	$[\infty,2^+,\infty]^+$	Ditropic
cm(h) (*×) c1	h: $[\infty^+,2^+,\infty]$	Monorhombic
cm(v) (*×) c1	v: $[\infty,2^+,\infty^+]$	Monorhombic
pgg (22×) p_g2_g	$[((\infty,2)^+)^{[2]}]$	Diglide
cmm (2*22) c2	$[\infty,2^+,\infty]$	Dirhombic

Parallelogrammatic (oblique)

IUC (Orb.) Geo	Domain Conway name
p1 (°) p$\bar{1}$	Monotropic
p2 (2222) p$\bar{2}$	Ditropic

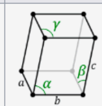
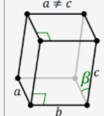
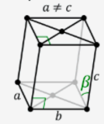
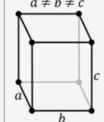
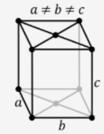
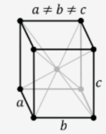
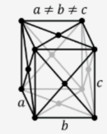
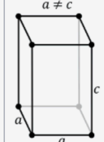
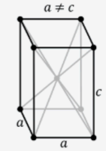
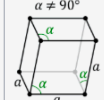
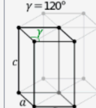
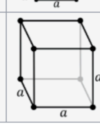
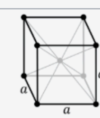
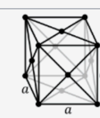
Hexagonal/Triangular
[6,3] / $[3^{[3]}]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) p$\bar{1}$		Monotropic
p2 (2222) p$\bar{2}$	$[6,3]^{\Delta}$	Ditropic
cmm (2*22) c2	$[6,3]^{⅄}$	Dirhombic
p3 (333) p$\bar{3}$	$[1^+,6,3^+]$ $[3^{[3]}]^+$	Tritropic
p3m1 (*333) p3	$[1^+,6,3]$ $[3^{[3]}]$	Triscopic
p31m (3*3) h3	$[6,3^+]$	Trigyro
p6 (632) p$\bar{6}$	$[6,3]^+$	Hexatropic
p6m (*632) p6	[6,3]	Hexascopic

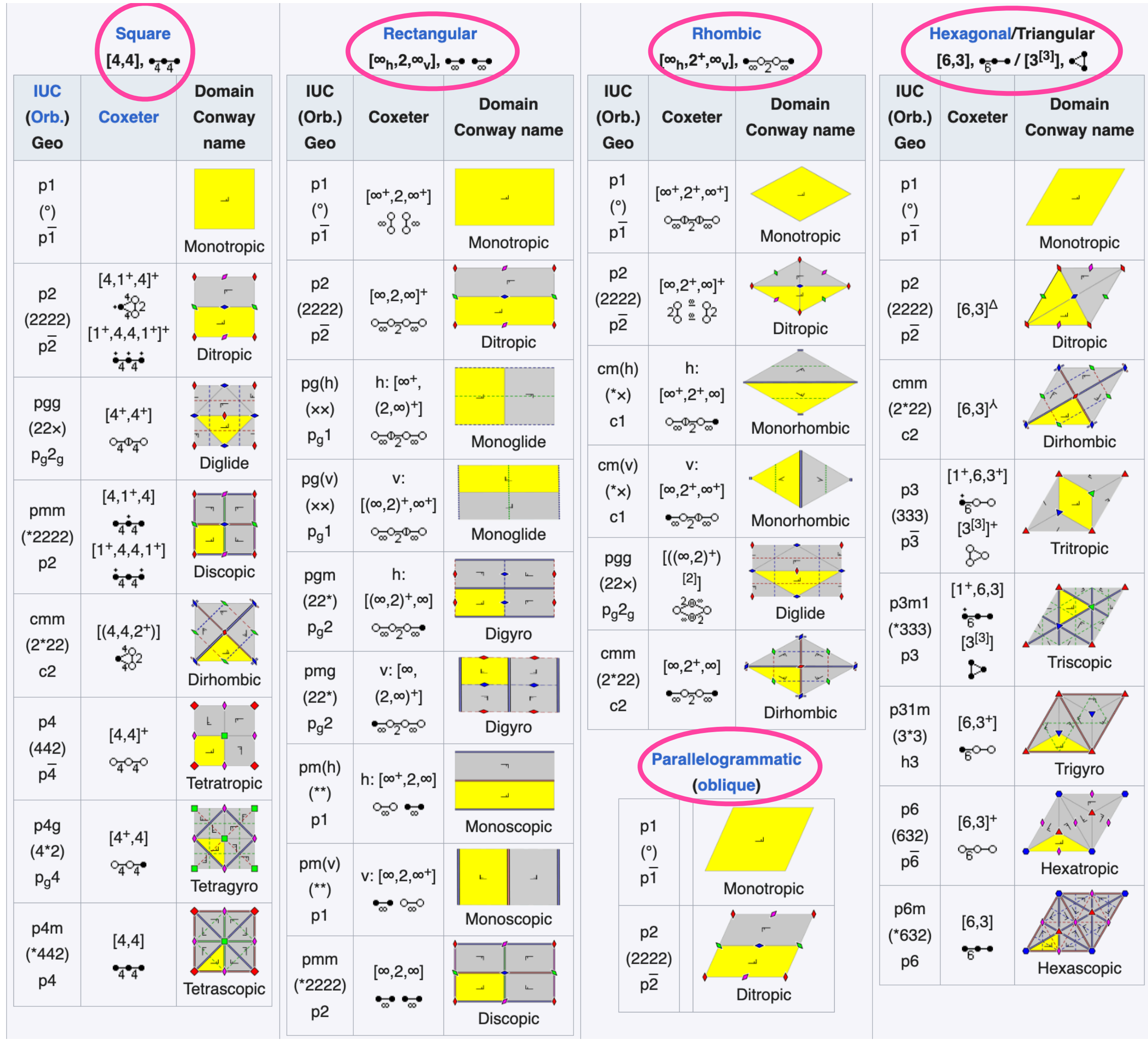
Bravais Lattices

- Mathematical definition: an infinite arrangement of points in space such that the lattice looks exactly the same when viewed from any lattice point.
- In 3D, Bravais Lattices define the 14 different configurations into which atoms can be arranged in crystals.

14 3D Bravais Lattice Structures

Crystal Family	Lattice System	Schönflies	14 Bravais Lattices			
			Primitive (P)	Base-centered (C)	Body-centered (I)	Face-centered (F)
Triclinic		C_i				
Monoclinic		C_{2h}	$\beta \neq 90°$ $a \neq c$ 	$\beta \neq 90°$ $a \neq c$ 		
Orthorhombic		D_{2h}	$a \neq b \neq c$ 	$a \neq b \neq c$ 	$a \neq b \neq c$ 	$a \neq b \neq c$ 
Tetragonal		D_{4h}	$a \neq c$ 		$a \neq c$ 	
Hexagonal	Rhombohedral	D_{3d}	$\alpha \neq 90°$ 			
	Hexagonal	D_{6h}	$\gamma = 120°$ 			
Cubic		O_h				

17 Wallpaper Groups (2D)



Square $[4,4]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$		Monotropic
p2 (2222) $p\bar{2}$	$[4,1^+,4]^+$ $[1^+,4,4,1^+]^+$	Ditropic
pgg (22×) p_g2_g	$[4^+,4^+]$	Diglide
pmm (*2222) p2	$[4,1^+,4]$ $[1^+,4,4,1^+]$	Discopic
cmm (2*22) c2	$[(4,4,2^+)]$	Dirhombic
p4 (442) $p\bar{4}$	$[4,4]^+$	Tetratropic
p4g (4*2) p_g4	$[4^+,4]$	Tetragyro
p4m (*442) p4	$[4,4]$	Tetrascopic

Rectangular $[\infty_h,2,\infty_v]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$	$[\infty^+,2,\infty^+]$	Monotropic
p2 (2222) $p\bar{2}$	$[\infty,2,\infty]^+$	Ditropic
pg(h) (××) p_g1	h: $[\infty^+,(2,\infty)^+]$	Monoglide
pg(v) (××) p_g1	v: $[(\infty,2)^+,\infty^+]$	Monoglide
pgm (22*) p_g2	h: $[(\infty,2)^+,\infty]$	Digyro
pmg (22*) p_g2	v: $[\infty,(2,\infty)^+]$	Digyro
pm(h) (**) p1	h: $[\infty^+,2,\infty]$	Monoscopic
pm(v) (**) p1	v: $[\infty,2,\infty^+]$	Monoscopic
pmm (*2222) p2	$[\infty,2,\infty]$	Discopic

Rhombic $[\infty_h,2^+,\infty_v]$

IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$	$[\infty^+,2^+,\infty^+]$	Monotropic
p2 (2222) $p\bar{2}$	$[\infty,2^+,\infty]^+$	Ditropic
cm(h) (*×) c1	h: $[\infty^+,2^+,\infty]$	Monorhombic
cm(v) (*×) c1	v: $[\infty,2^+,\infty^+]$	Monorhombic
pgg (22×) p_g2_g	$[((\infty,2)^+)^{[2]}]$	Diglide
cmm (2*22) c2	$[\infty,2^+,\infty]$	Dirhombic

Parallelogrammatic (oblique)

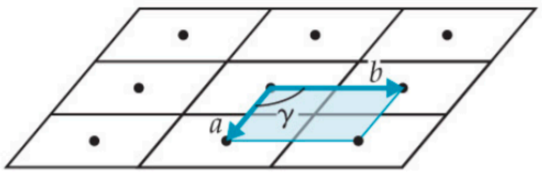
IUC (Orb.) Geo	Domain Conway name
p1 (°) $p\bar{1}$	Monotropic
p2 (2222) $p\bar{2}$	Ditropic

Hexagonal/Triangular $[6,3]$ / $[3^{[3]}]$

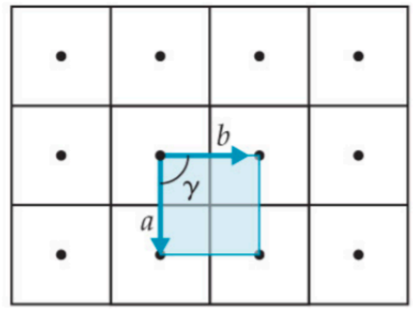
IUC (Orb.) Geo	Coxeter	Domain Conway name
p1 (°) $p\bar{1}$		Monotropic
p2 (2222) $p\bar{2}$	$[6,3]^\Delta$	Ditropic
cmm (2*22) c2	$[6,3]^⅄$	Dirhombic
p3 (333) $p\bar{3}$	$[1^+,6,3^+]$ $[3^{[3]}]^+$	Tritropic
p3m1 (*333) p3	$[1^+,6,3]$ $[3^{[3]}]$	Triscopic
p31m (3*3) h3	$[6,3^+]$	Trigyro
p6 (632) $p\bar{6}$	$[6,3]^+$	Hexatropic
p6m (*632) p6	$[6,3]$	Hexascopic

https://en.wikipedia.org/wiki/Wallpaper_group

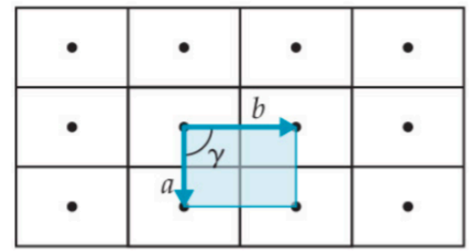
5 2D Bravais Lattice Structures



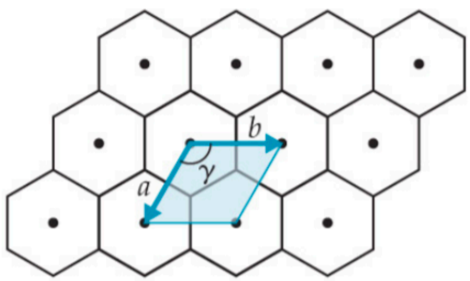
Oblique lattice ($a \neq b$, γ = arbitrary)



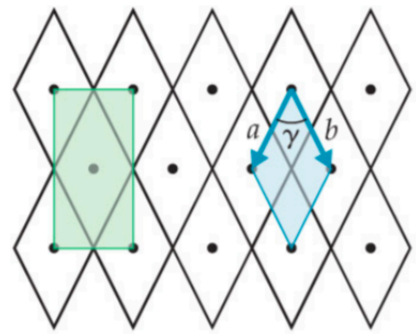
Square lattice ($a = b$, $\gamma = 90°$)



Rectangular lattice ($a \neq b$, $\gamma = 90°$)



Hexagonal lattice ($a = b$, $\gamma = 120°$)

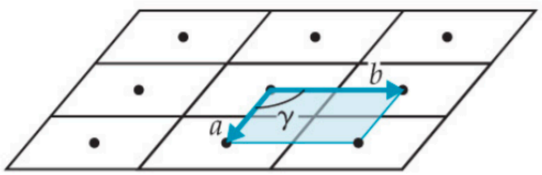


Rhombic lattice ($a = b$, γ = arbitrary)
Centered rectangular lattice

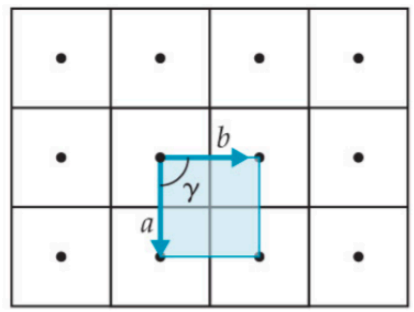
Bravais Lattice Structures

Any periodic 2D tiling maps to one of these 5 fundamental lattice structures.

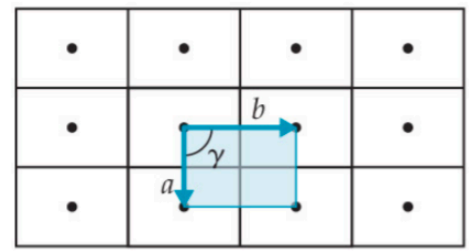
5 2D Bravais Lattice Structures



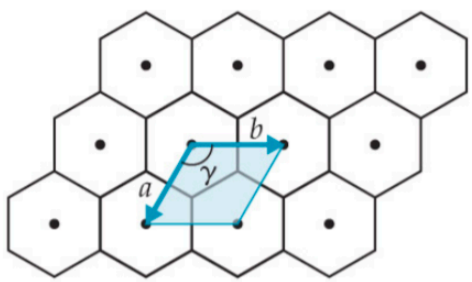
Oblique lattice ($a \neq b$, γ = arbitrary)



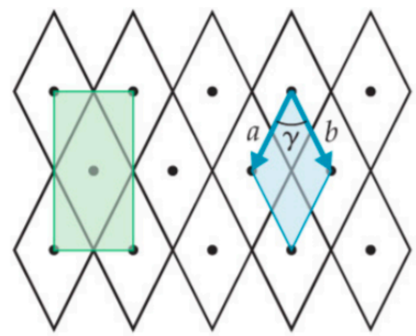
Square lattice ($a = b$, $\gamma = 90°$)



Rectangular lattice ($a \neq b$, $\gamma = 90°$)

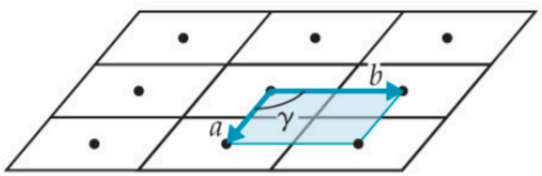


Hexagonal lattice ($a = b$, $\gamma = 120°$)

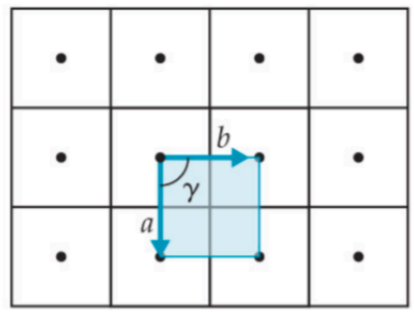


Rhombic lattice ($a = b$, γ = arbitrary)
Centered rectangular lattice

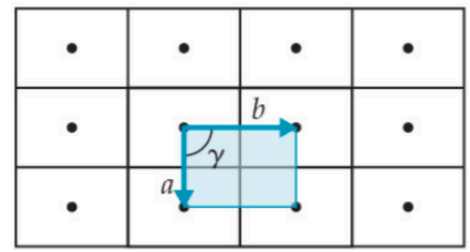
Note that they're all related



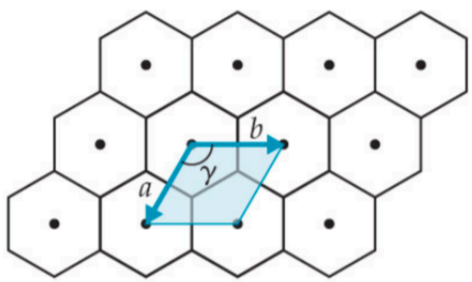
Oblique lattice ($a \neq b$, γ = arbitrary)



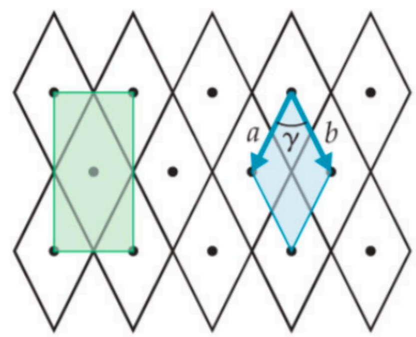
Square lattice ($a = b$, $\gamma = 90°$)



Rectangular lattice ($a \neq b$, $\gamma = 90°$)



Hexagonal lattice ($a = b$, $\gamma = 120°$)



Rhombic lattice ($a = b$, γ = arbitrary)
Centered rectangular lattice

Thank you!

CS 491 and 591
Professor: Leah Buechley
https://handandmachine.cs.unm.edu/classes/Computational_Fabrication_Spring2021/